

Numerical steepest descent

"pain-free numerical integration of oscillatory analytic fcn's."

Warm-up = quadrature, "inf" trap rule.



$f: \mathbb{R} \rightarrow \mathbb{C}$ $\int_{-\infty}^{\infty} f(x) dx \approx h \sum_{j \in \mathbb{Z}} f(x_j)$ $x_j = hj$

blow error scale w/ h?

$f'(0) \approx \sum_{m \in \mathbb{Z}} f\left(\frac{2\pi}{h} m\right)$ // Poisson summ formula

$2\psi(x-h) = \frac{1}{h} \sum_m \psi\left(\frac{x-h}{h} + m\right)$
 $i h \sum_j f(hj) = \sum_m f\left(\frac{2\pi}{h} m\right)$

so error $E_h[f] = \left| \sum_{m \neq 0} f\left(\frac{2\pi}{h} m\right) \right|$

take diff to get error. where $f(k) := \int f(x) e^{ikx} dx$

So if f bandlimited out to almost twice Nyquist, exact!

A) gaussian $f(x) = e^{-x^2/2} \leftrightarrow f(k) = \sqrt{2\pi} e^{-k^2/2}$

$E_h \approx \sqrt{4\pi} \cdot 2 e^{-\frac{1}{2} \left(\frac{2\pi}{h}\right)^2} \approx 5e^{-20/h^2}$ abs.

(plot it & compare)

h = 0.8 enough for (9-13) rel. err., but $f(8) \approx 10^{-14}$

(Show $e^{-x^2/2}$ in \mathbb{C})

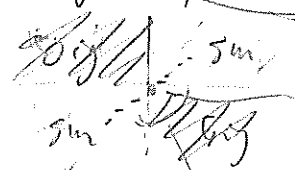
3) Osc. fcn's. $g(x) = \frac{1}{1+x^2} \cos(30(x-1)^2)$
 seek $I = \int_{-\infty}^{\infty} g(x) dx$
 slow amp. func. large phase func. real part

(plot) notice it's oscillatory check ✓
 ie $j = -10, -9, \dots, 10$ of these

$f(z) = \frac{1}{1+z^2} e^{i 30(z-1)^2}$ form $a(x) e^{i b(x)}$

analytic: plot in \mathbb{C}

note CTF in exp is similar form, radially.



stat. phase ph $x_0 = 1$: where $b'(x_0) = 0$

locally like gaussian rot. by $\pi/4$. : what scale? $|b''(x_0)| = 60$ of $\left(\frac{x}{2}\right)'' = 1$ for std. Gauss

so zoom in by $\sqrt{60}$
 so choose $h \leftarrow \frac{e^{+i\pi/4}}{\sqrt{60}} h$

chk conv ✓ $1e-12$

Closing: can do multi stat phase pb, change var to handle $\rightarrow \infty$ curved (analytic!) contours. or Gauss-Legendre on straight segments. slow JCP.