Multivariate Newton Interpolation

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In scientific computing, the problem of interpolating a multivariate function $f : \mathbb{R}^m \longrightarrow \mathbb{R}$, $m \in \mathbb{N}$, is ubiquitous. Because of their simple differentiation and integration, as well as their pleasant vector space structure, real polynomials $Q \in \Pi_{m,n}$ in m variables of degree $\deg(Q) \leq n$, $m, n \in \mathbb{N}$, are a standard choice as interpolants. Though, classical 1D–interpolation formulations as *Newton & Lagrange Interpolation* and their numerical solvers as the 1D–*divided difference scheme (DDS)* are known since the 18th century, no generalization of these methods has been established so far. This is at least partly due to the fact that an efficiently computable generator of *unisolvent nodes* $P_{m,n} \subseteq \mathbb{R}^m$, i.e., nodes that fix the interpolant $Q_{m,n,f} \in \Pi_{m,n}$ by requiring $Q_{m,n,f}(p) = f(p)$, $\forall p \in P_{m,n}$, was not known for the general multi–dimensional case.

Recently, we used concepts of *algebraic geometry* in order to establish an suitable generator of unisolvent nodes. The thereby obtained mathematical foundations allowed us to formulate a *multivariate divided difference scheme* that, analogously to the 1D case, computes the interpolant $Q_{m,n,f}$. In fact, all of the excellent runtime and accuracy performances of the 1D–DDS were preserved by our generalization. Moreover, the approximation error estimates known in 1D, naturally translate to the multi-dimensional case.

We aim to present this approach and discuss further developments and applications. In fact, our provided notion of unisolvent nodes goes beyond the requirements and perspectives of polynomial Newton interpolation. Therefore, we expect many computational schemes to benefit from our concepts including *fast Fourier transform, fast multi pole methods, spherical harmonics, numerical PDE solvers, non-linear optimization, linear/polynomial regression, spectral analysis, adaptive sampling, Bayesian inference.*

Related Publications

M. Hecht, K. B. Hoffmann, B. L. Cheeseman, and I. F. Sbalzarini, "Multivariate Newton interpolation," *arXiv preprint arXiv:1812.04256.*

M. Hecht and I. F. Sbalzarini, *Intelligent Computing, Proceedings of the 2018 Computing Conference, Volume 2.* Springer, Cham, 2018, vol. 2, ch. Fast Interpolation and Fourier Transform in High-Dimensional Spaces, pp. 53–75.

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