Math and Music

Developed by Megan Martinez and Alex Barnett in conjunction with Ilene Kanoff

For questions or comments, feel free to contact Megan Martinez at megan.ann.martinez [at] gmail.com

Overview

This lesson is an exploration into the mathematical world of music. Students will learn about the relationship between pitch, frequency, and period. Students will then apply their knowledge by constructing their own pan pipes.

Objectives

In this lesson, students will:

- *Learn to record sounds in Audacity and calculate the period and frequency of those sounds.*
- *Explore the relationship between period, frequency, and pitch.*
- *Understand how pipe length (one end closed and one end open) affects pitch and be able to compute precisely how length determines frequency and vice versa.*
- *Apply mathematical knowledge of the above to construct a precisely pitched pan pipe.*

Background *This activity can be used for many different levels, but ideally students will have a very basic level of algebra skills. They need to be able to apply formulas.*

Materials

- *Wide Straws (1/2" diameter)*
- *Clay (Modeling clay works well)*
- *Alcohol wipes (for disinfecting instruments between performers)*
- *Computers that have been installed with Audacity. This is a free program available at: http://audacity.sourceforge.net/*
- *Relevant worksheets:* **The Periods and Frequencies of Sound**, **Periods and Frequencies Practice**, **Math and Music Practice**

Lesson Plan

This is a suggested lesson plan. The material can be cut down or expanded as the user sees fit.

Part I - Audacity, Periods, and Frequencies

KEY IDEAS FOR PART I:

- The definition of the period and frequency of a sound
- How to measure the period of a sound using Audacity
- The equation that relates frequency and period:

$$\frac{1}{\text{Period}} = \text{Frequency}$$

SET-UP:

1. Have a computer ready with Audacity that can be displayed to show students the process or print out the directions in **Appendix A**
2. Split students into groups that they will be in for the duration of the unit.
3. Each group needs to have access to a computer.
4. Print out copies of the worksheets: **The Periods and Frequencies of Sound** and **Periods and Frequencies Practice**

PROCEDURE

1. Set-up the activities for the day with a quick discussion on sound. If you have a tuning fork, this would be a good opportunity to use it. Make sure students understand that sound is created by vibrations in the air. When we record sounds, it's these vibrations that we are keeping track of.
2. Introduce students to Audacity. Make sure they have their own program set to the right settings. (See Appendix A for how to use Audacity)
   (a) Record a steady tone on Audacity. You can use your voice to do this - make sure you keep the pitch steady and record the sound for long enough.
   (b) Zoom in on the recorded sound so that students can see the sound wave. Ask them questions about the graph: What does the x-axis represent? (Time.); What do you notice about the shape of the graph? (The graph forms a repeating pattern). They may ask about the y-axis - this represents air pressure.

3. Introduce two definitions:

   (a) **Period**: The amount of time it takes for a sound wave to repeat one cycle of its pattern.

      i. At this point you can refer back to Audacity and teach students how to calculate the period of a sound they make.

   (b) **Frequency**: The number of repetitions of a sound wave's pattern per second.

      i. Have students think together to come up with the equation for frequency.

      ii. The equation for this is

$$\frac{1}{\text{Period}} = \text{Frequency}$$

      . The unit for frequency is Hertz (Hz).

4. Distribute the worksheet, **The Periods and Frequencies of Sound**. Students should understand the following actions in Audacity before they are set loose on the exploration:

   (a) Use "record" and "stop" buttons to record on Audacity

   (b) Using the zoom function and the mouse, calculate how long it takes for the sound wave to complete 10 periods.

   (c) Divide this time by 10 to get the actual period

   (d) Use the formula

$$\frac{1}{\text{Period}} = \text{Frequency}$$

   to get the frequency.

5. After students have completed problem 1 on the worksheet, go through the procedure as a class to make sure everyone is on the same page.

6. After students have completed 2 (3 is just for fun and those who go faster), bring the class back together and talk about the results. Some correct observations are the following:

   (a) The larger the frequency, the higher the pitch.

   (b) The smaller the frequency, the lower the pitch.

   (c) The smaller the period, the higher the pitch.

   (d) The larger the period, the lower the pitch.

7. Assign the homework **Periods and Frequencies Practice**.

Part II - Frequency, Note Ratios, and Pipe Lengths

KEY IDEAS FOR PART II:

- If you multiply the frequency of a note by 2, you will get the note one octave higher.
- if you multiply the frequency of a note by 3, you will get the note that is an octave and a fifth higher.
- Playing a pipe with one end closed and one end open follows the following equation (where the Length is in meters):

$$\text{Frequency} = \frac{340}{4 \cdot \text{Length}}$$

SET-UP:

- Have a signal generator set up to produce pitches. If you don't have a signal generator laying around, you can use online pitch generators to get the same effect. Ideally a pitch generator that comes with a slider so you can continuously change the frequency without typing numbers in. Here are some free online tone generators:
  - http://plasticity.szynalski.com/tone-generator.htm - this is a continuous slider tone generator. It works only on Firefox.
  - http://onlinetonegenerator.com/ - This does not have a continuous slider, but it is easy to use and reliable.

  You need two different sources of pitches (this could mean two computers, or simply two windows open with the tone generators).
- (Optional) A tuning fork pitched at A 440 would be useful in the exercises for the day.
- Have two straws constructed (as in the pan pipe project) with one end open and the other end sealed with clay. Having one pitched A 440 and the other pitched A 880 is fun, since then students can compare the lengths of the straws (one is half the length of the other)

PROCEDURE

1. Go over the previous day's homework, **Periods and Frequencies Practice**. The answers are:
   1. (a) Period = 2, Frequency = 1/2
      (b) Period = 3, Frequency = 1/3
   2. Period = $\frac{1}{250} = .004$
   3. Period = $\frac{1}{1247} = .000802$
   4. The sound is too low for human ears to hear.

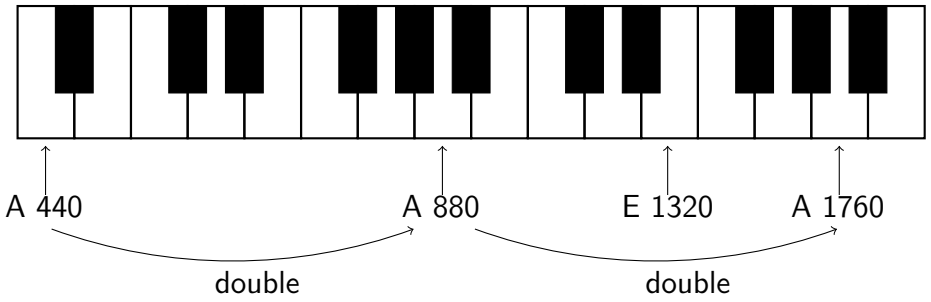
2. The first activity of the day will be to get the concept of the ratios between the frequencies of different notes.

   (a) Play an A 440 on one of your available pitch generators (tuning fork, computer, or other). Now use your second pitch generator (ideally one with a way to "slide" the frequency without typing in numbers). Set this one to A 440 also and start sliding the frequency higher. Ask students to raise their hands when the pitch sounds the same again - when you've reached the note precisely one octave higher (this should happen at 880 Hz, though some may think it happens at 660 Hz, since this is the perfect fifth). Stop on that frequency.

   (b) Ask students what they notice about the two frequencies that are playing. (The higher frequency is twice the lower frequency.)

   (c) Now ask students to talk in groups to try and figure out the following: What will be the frequency if you go up one more octave? (They will likely come back with two different answers: 1320 Hz or 1760 Hz) Test the conjectures with your equipment. The correct answer is 1760 Hz.

   (d) On the board write a diagram similar to the following:



   You can note that 1320 Hz is actually the E between A 880 and A 1760.

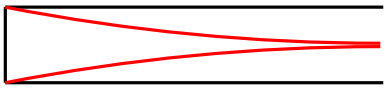
   (e) Ask groups to figure out in pairs the following: What is the frequency of the E between A 440 and A 880? (They can most easily get the answer by dividing 1320 by 2, so the answer is 660 Hz. There are other methods.)

3. The second activity is to get the concept of how pipe length relates to frequency.

   (a) First demonstrate the pipes you brought to class. Make it clear that we are working with pipes that have one end open and the other end sealed (You get a different equation if both ends are open). Ask what they notice about the pipes and the sound that they make (The one that is shorter produces a higher note). If you have pipes that are an octave apart you can also ask about the relationships between the lengths (One is half the length of the other).

   (b) At this point you want to talk about what is happening with the air in the pipe as you blow into it. Here is a good website that talks about the concepts: http://hyperphysics.phy-astr.gsu.edu/hbase/waves/clocol.html

When you blow into the open end of the pipe, the sound wave is reflected when it hits the closed end and simply bounced back when it hits the open end. As a result, the wave will travel the length of the pipe 4 times before it repeats and the period of the sound wave is the amount of time it takes to travel the pipe four times.



(c) So, we need the speed of sound to come up with an equation that relates frequency and pipe length. We will estimate it to be 340 m/s. Then we get

$$\text{Period} = \frac{4 \cdot \text{Length}}{340}$$

Using what we know about period and frequency we get

$$\text{Frequency} = \frac{340}{4 \cdot \text{Length}}$$

And using a bit of algebra we get

$$\text{Length} = \frac{340}{4 \cdot \text{Frequency}}$$

(d) Assign students the worksheet **Frequency and Pipe Length** for homework.

Part III - Constructing Pan Pipes

KEY IDEAS FOR PART III

- Apply the knowledge from parts I and II into real working experience through construction of pan pipes.

SET-UP

1. Have available for each group: computer with Audacity, straws, tape, rulers, scissors, and clay.

PROCEDURE

1. Go over the homework. Answers are:
    1. 256 Hz
    2. (a) 0.1288 m = 12.88 cm
       (b) 0.0483 m = 4.83 cm
    3. 488.5057 Hz. Here, students have to be careful to convert the pipe length into meters.
2. After you have gone through the homework, introduce the project. Hand out the **Math and Music Project** sheet. There are a few things that should be emphasized when introducing the project:
    (a) Groups should work independently from other groups and there needs to be cooperations within groups.
    (b) Students are graded mainly on their documentation of work. If they don't have documentation, but have a well-made pan pipe, they won't get a very good grade.
    (c) Students are intended to experiment with the pipes. The teacher is not going to provide the answer. They need to rely on their knowledge of the math to create the instrument!
    (d) Pan pipes are played be blowing across the top of the instrument. Students that play the flute will have little trouble with this. For others, it may take a few tries to get the hang of.
3. You can decide how much in class time students require for the project. 2-3 class periods is likely appropriate. Upon completion of the project, hold a performance where groups can demonstrate their instrument!

Name:_____ Date:_____

The Periods and Frequencies of Sound

1. Using your voice, make a musical sound (it must be steady and long enough). Measure how long it takes for the sound wave to complete ten periods. Determine the period, then the frequency of this sound, using that information.

2. Record 2-3 different pitches (high, low, middle). Calculate the periods and frequencies of these sounds.

   (a) Make three mathematically relevant observations about the periods and frequencies of these sounds.

   (b) How do the sounds you made account for the different frequencies?

3. Make and record a sound whose graph is not periodic.

Name:_____ Date:_____

Periods and Frequencies Practice

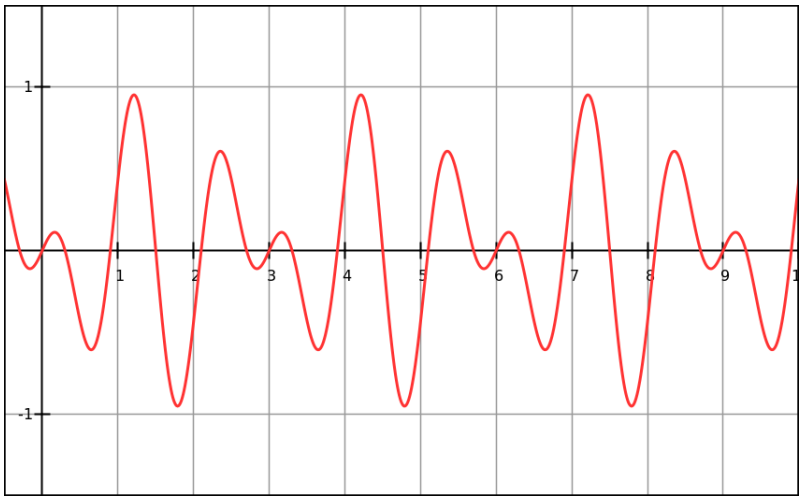
1. Determine the period and frequency of the waves in the following graphs:

(a)

Period = _____

Frequency = _____

(b)



Period = _____

Frequency = _____

2. A sound has the frequency of 250 Hz. What is the period of this sound?

3. You hit a tuning fork that produces a sound with frequency 1247 Hz. What is the period of the sound produced by the tuning fork?

4. You have another tuning fork (different from in problem 3) that vibrates 3 times a second (so has a frequency of 3 Hz). You can't hear this sound - why do you think this is?

Name:_____ Date:_____

Frequency and Pipe Length

1. You have a pipe that has length .332 m. What is its frequency?

2. For (a) and (b), you will need to first figure out the frequency of the note mentioned, then you will find the pipe length.

   (a) Find the pipe length for E between A (440 Hz) and A (880 Hz).

   (b) Find the pipe length for A two octaves above A (440 Hz).

3. You have a pipe that has length 17.4 cm. What is its frequency?

Name:_____ Date:_____

Math and Music Project

This is a project that is more about process than product. You are responsible for documenting your trials and errors in making your instrument.

The Project: You and your partner are responsible for creating a pan pipe that produces at least five different pitches.

1. Cut one of your straws to produce the frequency 440 Hz. Test your pitch on Audacity to make sure it is correct (Don't stick the clay into the closed end more than a few millimeters).

2. For the rest of the pitches, you can decide what frequency you want them to be, but you must determine how long the straws should be cut. You must document the math that you used to determine the length of your remaining straws. Note: **You have a maximum of 12 straws per group, so make sure you think out the math before you cut.**

3. Using Audacity, check the frequencies of each of your pipes to determine their accuracy. If a pipe does not meet your expectations, record why you believe your attempt didn't work, what you need to do differently, and construct a new pipe (or modify your current pipe). See table below for documenting this. **Failure to maintain accurate and detailed documentation will result in a lower grade.**

4. All of your final pitches must be recorded in Audacity and the frequency and period must be documented in the attached chart.

5. You may also check the tuning of your pan pipe by going to http://onlinetonegenerator.com/ and plugging in the frequency you believe your pipe is producing.

Note Schemes

There are a number of different scales in music! Here are some five note scales or songs. Use these schemes for your pan pipes, or create your own!

Blues Scale = A, C, D, D♯, E (This is used in Jazz)
Pentatonic Scale = A, B, D, E, F♯ (used in a lot of traditional music all around the world)
Ode to Joy, When the Saints Go Marching in, Frere Jacques: A, B, C♯, D, E
Nobody Knows the Trouble I've Seen (6 notes): A, B, D, E, F♯, A (octave higher)

Table of Notes and Frequencies

Note	Frequency
A	440
A♯	466
B	494
C	523
C♯	554
D	587
D♯	622
E	659
F	698
F♯	740
G	784
G♯	830
A	880

If you would like to use other pitches, use the ratio to figure out their frequencies.

This exact table must be put on Google Docs and shared with your teacher.

Desired Note/Frequency	Length Used Show your work!	Measured Frequency	Did your attempt work? Explain what must be done differently if it didn't

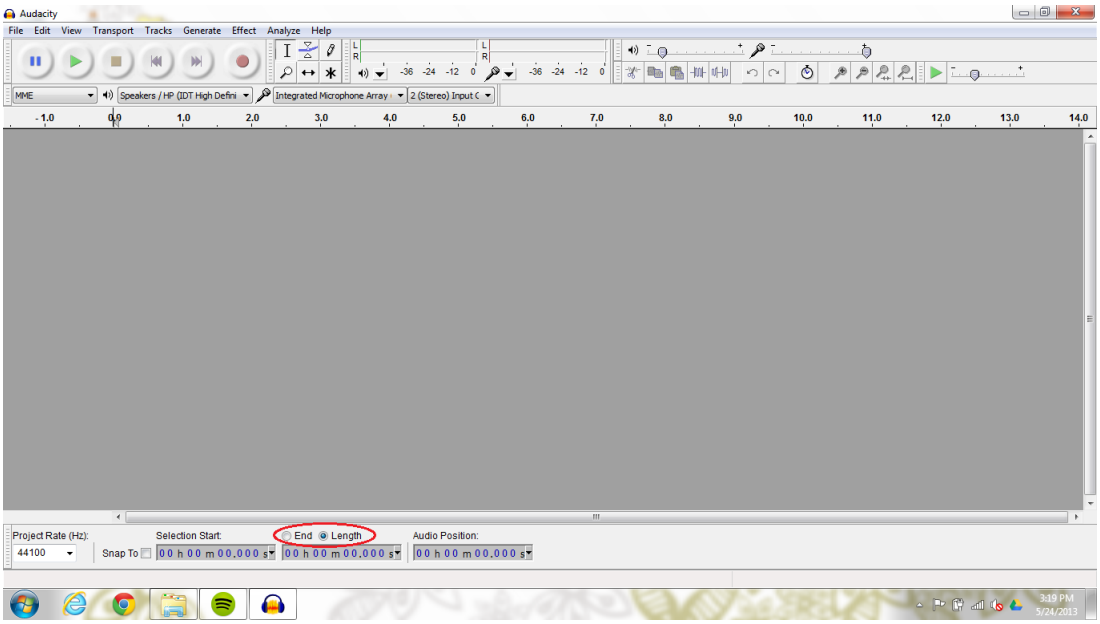
Grading Rubric

A	• 5 pipes made correctly • 5 frequencies computed without error • Table complete on Google Docs with correct calculations and no information missing (this includes fixing pipes that were off) • Group worked cooperatively and independently from other groups
B	• 4 pipes made correctly • 4 frequencies computed without error • Table complete on Google Docs with correct calculations and 1-2 pieces of information missing (this includes fixing pipes that were off) • Group worked cooperatively and independently from other groups
C	• 3 pipes made correctly • 3 frequencies computed without error • Table complete on Google Docs with correct calculations and 1-2 pieces of information missing (this includes fixing pipes that were off) • Group worked fairly cooperatively and independently from other groups
D	• 2 pipes made correctly • 2 frequencies computed without error • Table complete on Google Docs with correct calculations and 3-4 pieces of information missing (this includes fixing pipes that were off) • Group did not work cooperatively and independently from other groups
F	• 1 pipe made correctly • 1 frequency computed without error • Table incomplete on Google Docs • Group did not work cooperatively and independently from other groups

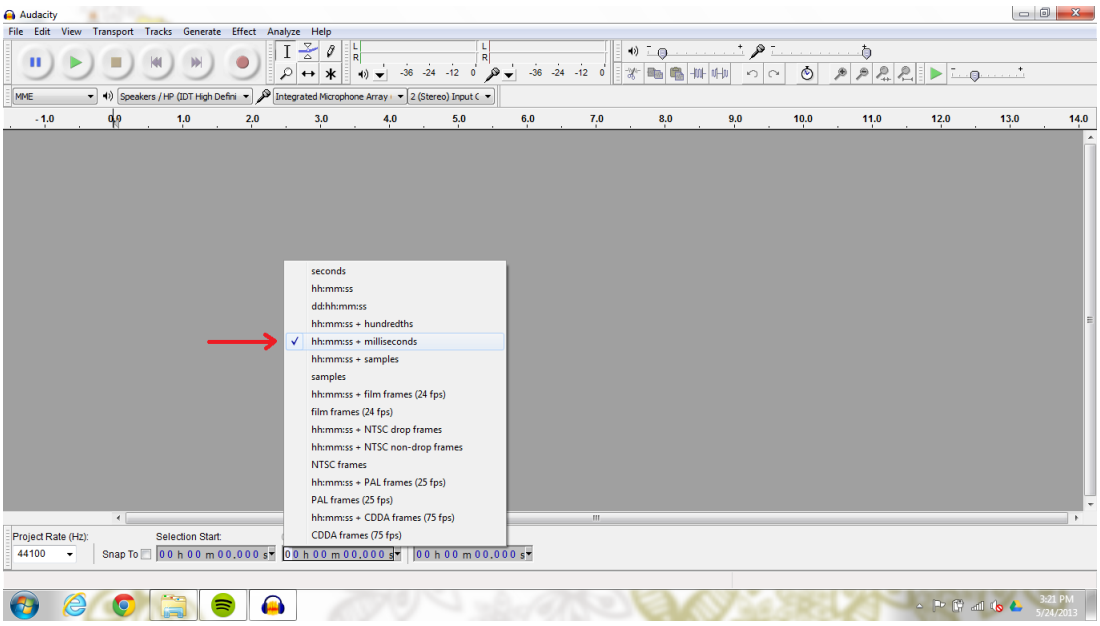
Appendix A

This is a guide on how to use Audacity to measure the period of a sound.

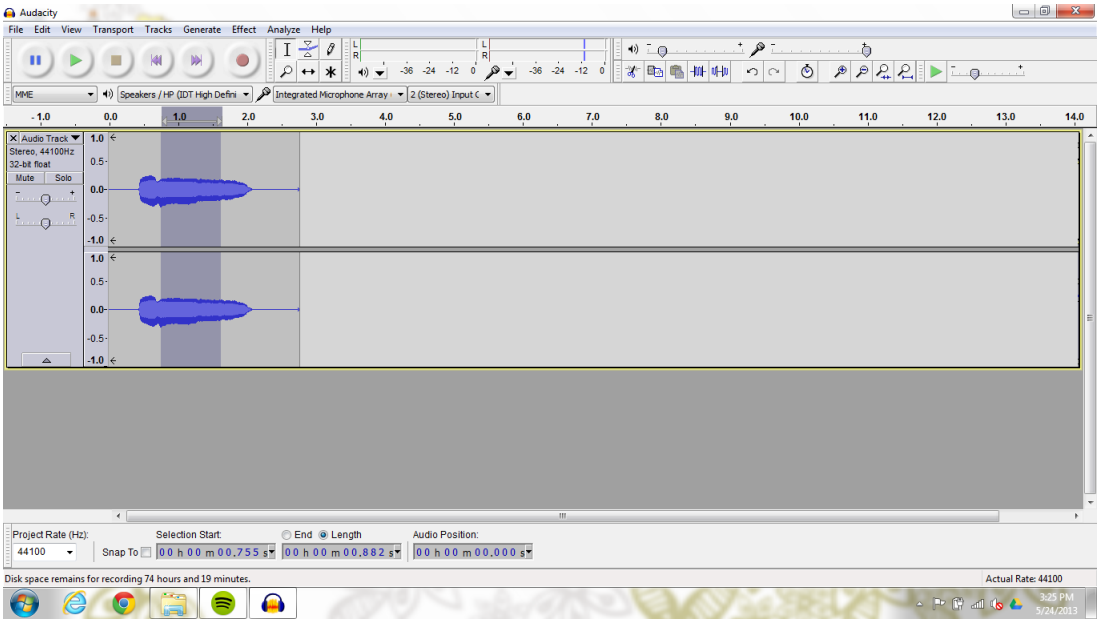
Step 1: Open Audacity and make sure you have the measurement setting on the bottom set to "Length."



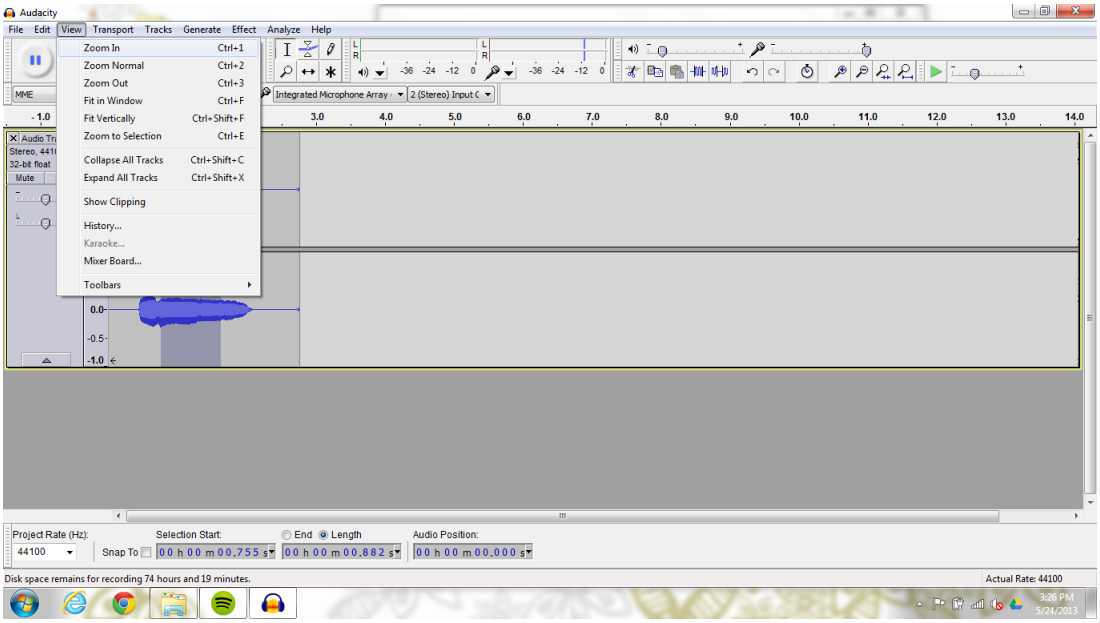
Step 2: Set the measurement units to "hh:mm:ss + milliseconds."



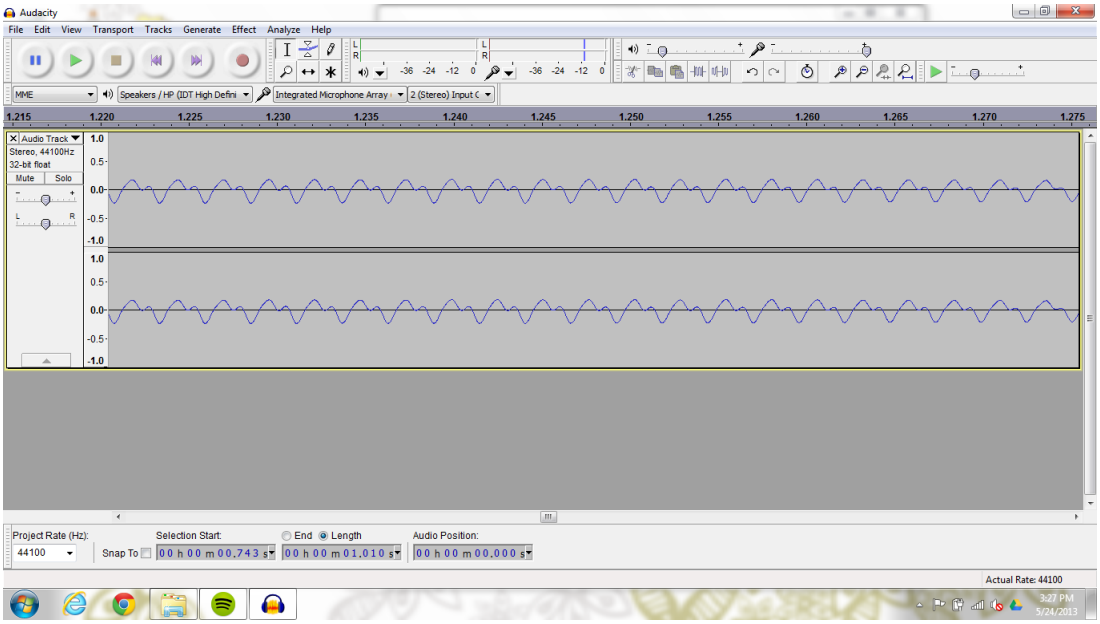
Step 3: Press the record button in the upper left side of the screen. Make a long, steady sound and press the stop button. With your mouse, select the middle of the recorded sound.



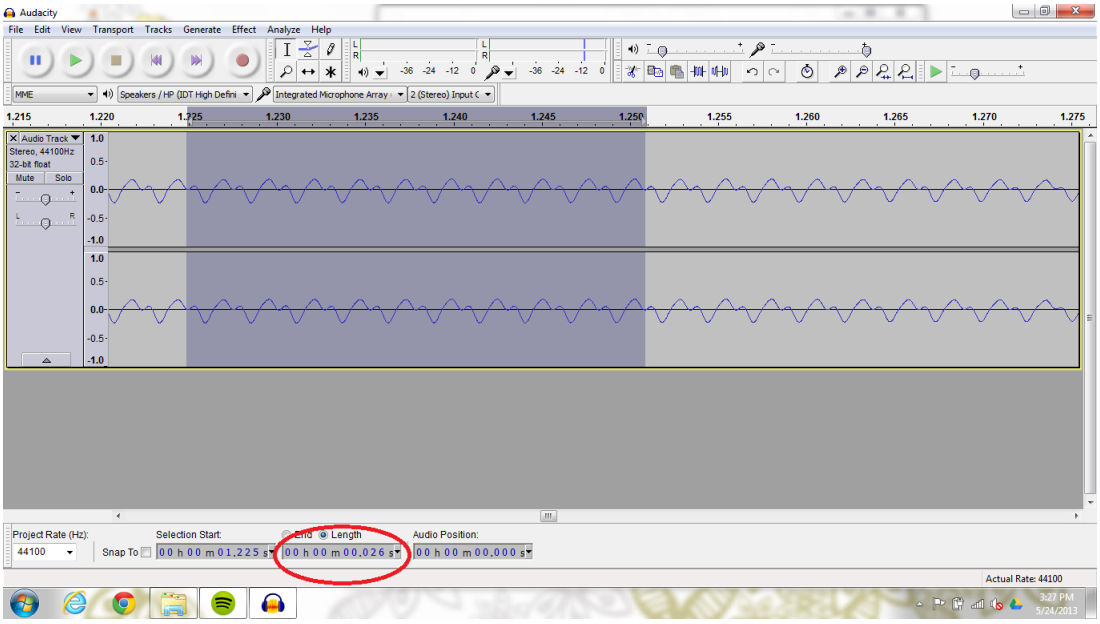
Step 4: Zoom in on the display. You can use the quick keys to do this, press "Ctrl" and "1."



Step 5: Zoom in until you can see the specific sound waves.



Step 6: With your mouse, highlight 10 repetitions of the period. Read off the amount of time it took for the sound wave to complete the 10 periods. Then divide this number by 10 to get the actual period.



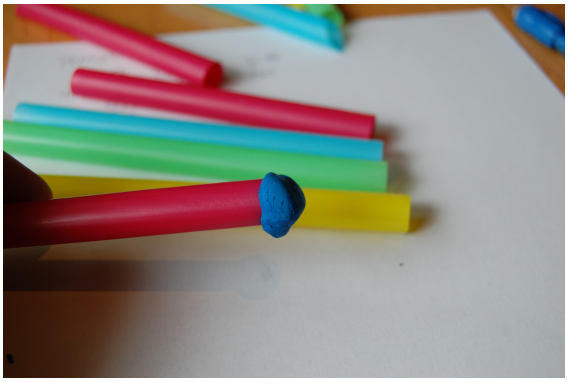
Appendix B

This guide details how to construct a pan pipe with the following materials: Wide straws, clay, tape, scissors, ruler, pencil, and paper.

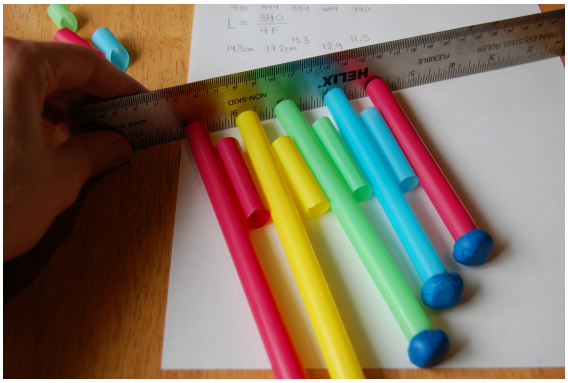
Step 1: Make the necessary calculations for the lengths of the pipes.

Step 2: Cut off the ends of the straws. The straws pictured had points on one end - make sure to cut these off. You want the ends of your straws to be flat on each end.

Step 3: Seal one end of the straws with clay. Make sure not to stick the clay in too far and make sure it is completely sealed, or the pipe will have trouble sounding.



Step 4: Use the straw ends that were cut off to act as spacers between your pipes. Aline the tops of the pipes you intend to play.



Step 5: Tape the straws together so you have a portable instrument