Senior Honors Class: Mathematical Wave Dynamics

Spring 2004 — Oliver Bühler & Alexander Barnett

Problem Set #2. Due February 17, 2004.

You will use MATLAB to plot different aspects of the 1D wave scattering problem from the material interface (lecture 3). This homework is sufficiently complicated that you won't want to leave it to the night before it's due!

1. We'll investigate transmission and reflection as a function of material wavenumber ratio $\alpha \equiv k'/k = c/c'$. [1] Plot graphs showing four quantities as a function of α: amplitude reflection and transmission coefficients a and b, and power reflection and transmission coefficients $R = |a|^2$ and $T = \alpha|b|^2$. Notice R = reflected flux / incoming flux, and T = transmitted flux / incoming flux. (These follow from notes in lecture 3). You should test (by eye is fine) that $R + T = 1$, *i.e.* energy is conserved, for the curves you get. Make sure your axes are correctly labelled, and the curves are also labelled (`legend` command).

   (a) Plot linear axes (for both axes), in range $\alpha \in [0, 5]$ showing all four curves a, b, R, and T.

   (b) Plot logarithmic axes (for both axes), in range $\alpha \in [10^{-3}, 10^3]$ showing just two curves R and T.

   (c) When is no power reflected?

   (d) For $\alpha \ll 1$, what power of α does T tend to follow? (Eyeball your log plot)

   (e) For $\alpha \gg 1$, what power of α does T tend to follow?

   NB You can find MATLAB help links on our course website. To get you started, you will need to make an array of α values, using *e.g.* `alpha = logspace(-3, 3, 100);` As always when using computers, start by doing something simpler first, *e.g.* make a single curve on a graph. Ask us if stuck.

2. Let's fix a material ratio $\alpha = 5$ (lighter going to heavier rope), with $c = 1$, and incoming wavenumber $k = 1$. Plot the displacement field $u(x, t)$ as a function of x at fixed t. Remember u is real. Choose a sensible domain of x to show the wave on both sides of the interface. You will need to find a way to plot u which is defined by different functions for $x \le 0$ and $x \ge 0$. One way is by plotting twice on the same graph.

   (a) Plot a graph at $t = 0$. (Check by eye that u and u_x are continuous at $x = 0$).

   (b) Plot a graph at $t = \pi/2$.

   (c) [If time]. Make an animation of $u(x, t)$ by replotting many times as t is increased. You may find the `drawnow` and `pause` commands useful.

Submission: you can either print everything out, or email postscript graph answers and code to us. Either way, you must include the MATLAB commands you used (in a M-file is best).

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[1] As in lecture, we are holding tension $s' = s$ constant, but we most generally could write $\alpha = \sqrt{s'\mu'/s\mu} = z'/z$, the 'impedance' ratio. If you study sound or other physical waves, impedance is the correct quantity to consider.