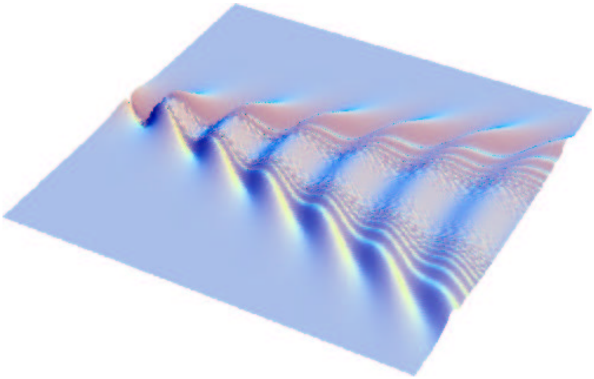
V63.0394: Senior Honors Course, Spring 2004
Mathematical Wave Dynamics

Instructors: **Oliver Buhler & Alexander Barnett**

Prerequisites: Calculus I, II and III. Tues, 2 to 5pm, WWH 1314.



ship's wake simulation

wave resonances of cavity

Description

Waves and wave propagation are essential features of the natural world and they lead to many puzzling questions. For instance, what produces mirages in the desert? Why is it that a storm over the ocean sets off a steady swell of small-amplitude waves, but an earthquake at the sea floor can release an enormous flood wave? Can you hear the shape of a drum? In response to these questions, a great deal of mathematics has been developed to understand and predict the dynamics of light or sound waves, the propagation of quantum (matter) waves, the vibration patterns of elastic bodies, or the peculiar nature of water waves. This course will give a guided tour of mathematical wave theory together with physical applications.

We will start with the classical wave equation and normal mode theory, which describes the vibration spectrum of elastic media, and also of trapped quantum particles. We will introduce the universal phenomena of diffraction and interference of travelling waves. Small-scale wave asymptotics then leads to Fermat's famous principle, which describes underwater sound propagation and light propagation in refractive media. We will then introduce the important concepts of wave dispersion and group velocity, which are vital to understand water waves and many other geophysical waves. This will lead to the ray theory of small-scale waves and its beautiful Hamiltonian structure, with applications such as beach surf and waves propagating on a vortex. We will also look at Sobolev's famous problem of dispersive waves in a rotating tank, which had implications for early spacecraft design, and which has recently been rediscovered in the study of the natural vibration of lakes. Finally, nonlinear waves can lead to new phenomena such as shock formation or solitary waves. We will look at simple examples such as traffic flow models or the celebrated KdV equation, which helped spawn the multimillion dollar industry of fiber optics. The course will also introduce some numerical techniques involved in wave calculations.

By about half way through the semester, each student will have chosen a topic for more intensive study. Typically this will involve numerical investigation of some wave problem on a computer (in which case programming experience is helpful), but a theoretical topic is also possible. They will then present their results to the rest of the class in the final week or two.

Grading: The final grade will be based on the project presentation, on a written report to be handed in at the end of the semester, and on two or so homework assignments given throughout the semester.

References: There is no basic text for the course. Notes will be provided in class, and a number of references will be on reserve in the library. Feynmann Lectures on Physics 1–3 has some great material. For the PDE content of the course, any basic text will suffice, such as Habermann's Elementary Applied Partial Differential Equations.

Other information

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Office hours 2–4pm Thursdays.
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Office hours 2–4pm Wednesdays.

Course website, containing resources and useful links (feel free to give suggestions to Alex):

`http://www.cims.nyu.edu/~barnett/waves`