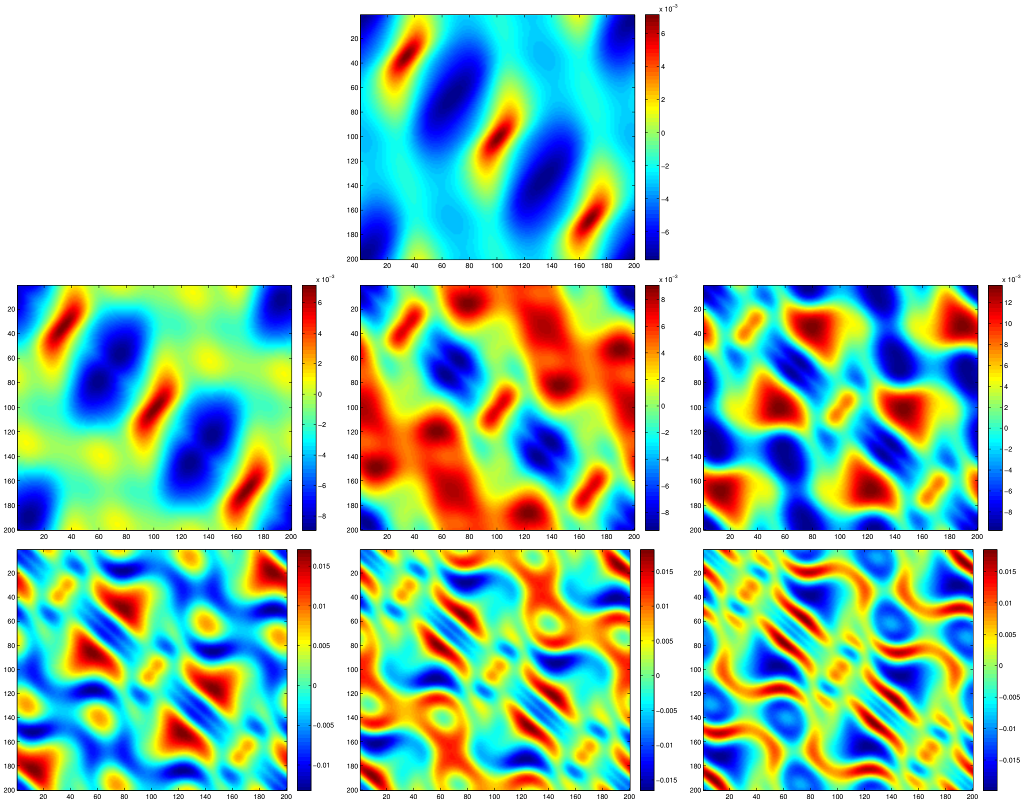
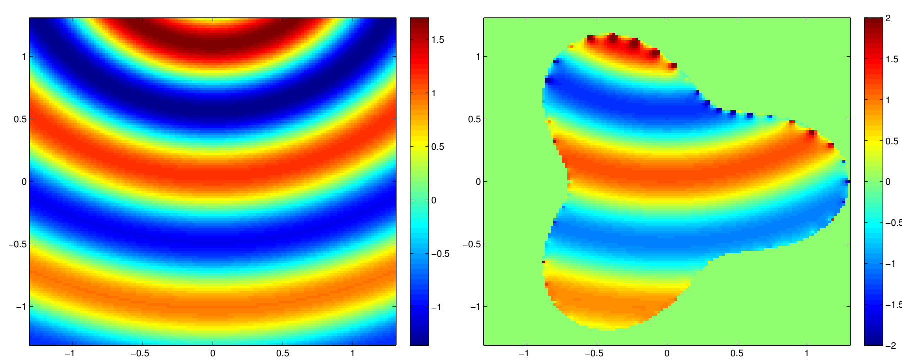
Brad Nelson
2/20/12
Math 126 HW 6

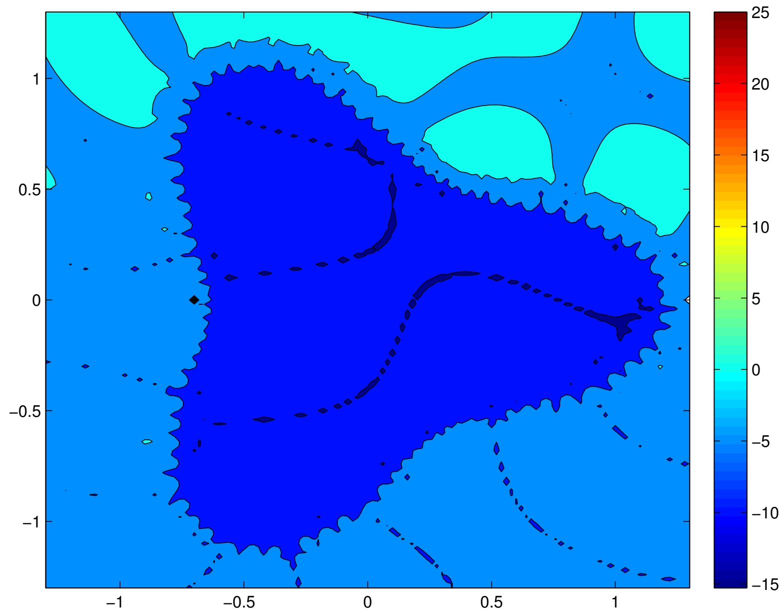
1. (a) Code: hlker.m hlkervis.m



Helmholtz DLP kernels: top: k=0 (Laplace equation); middle, left to right: k=1, k=2, k=3; bottom, left to right: k=4, k=5, k=6.

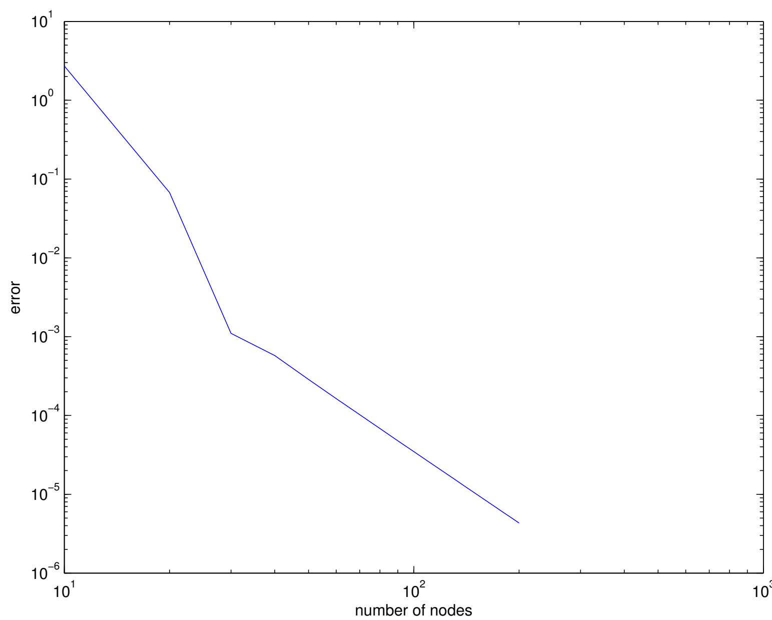
   (b) Code: bvphelmvis.m





Top left: actual solution. Top right: reconstructed solution inside boundary. Bottom: $\log_{10}$ error of reconstructed solution.

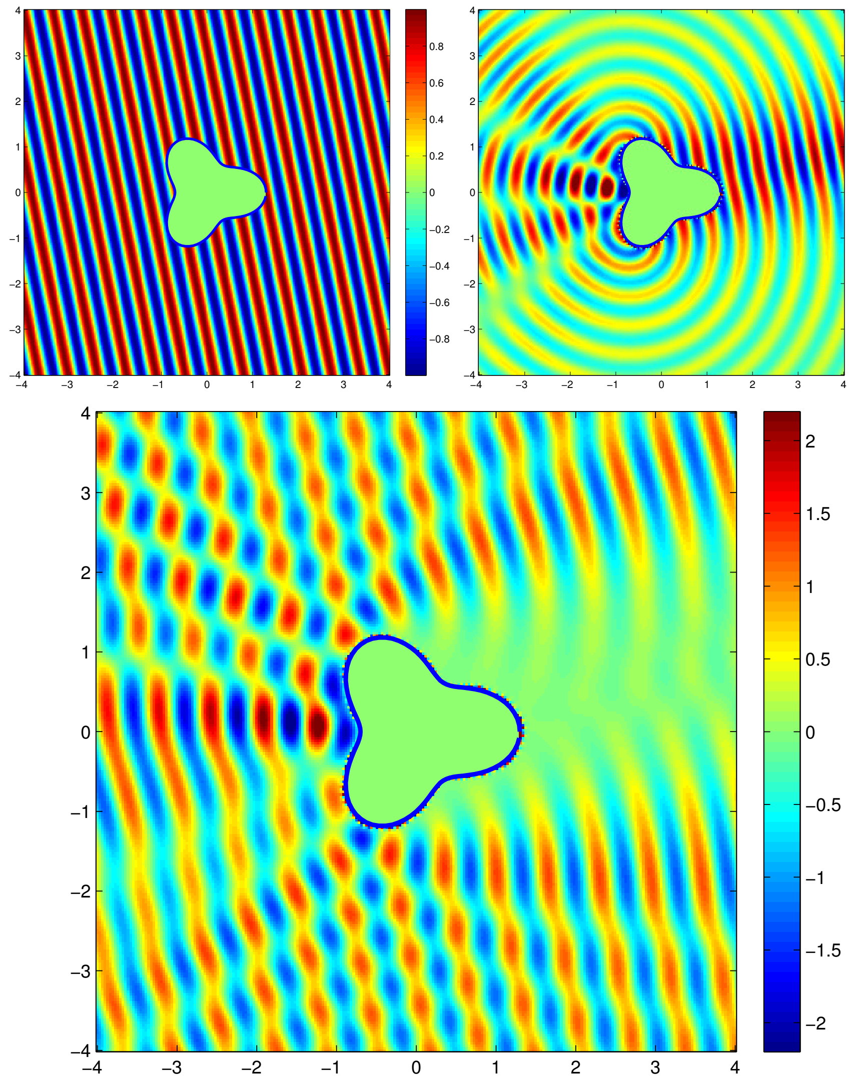
(c) Code: bvphelmerror.m



Error of reconstructed solution at $(0.2, 0.1)$, vs. N nodes.

The error converges algebraically, with order -1.48. This is not as good as the Laplace equation, which had exponential convergence. This suggests that the Helmholtz DLP kernel is not as smooth, which we can see in part (a).

2. (a) Code: scathelm.m

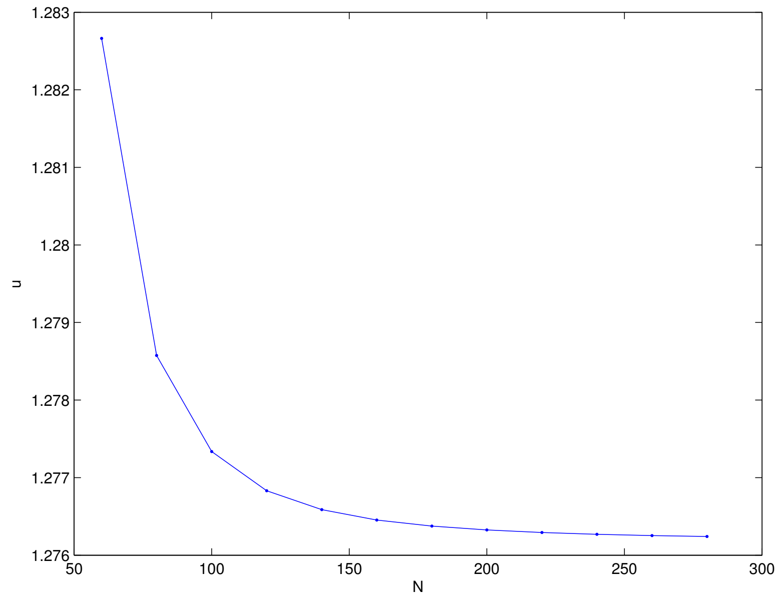


Top left: incident wave u^i. Top right: scattered wave u^s. Bottom: total wave $u = u^i + u^s$.

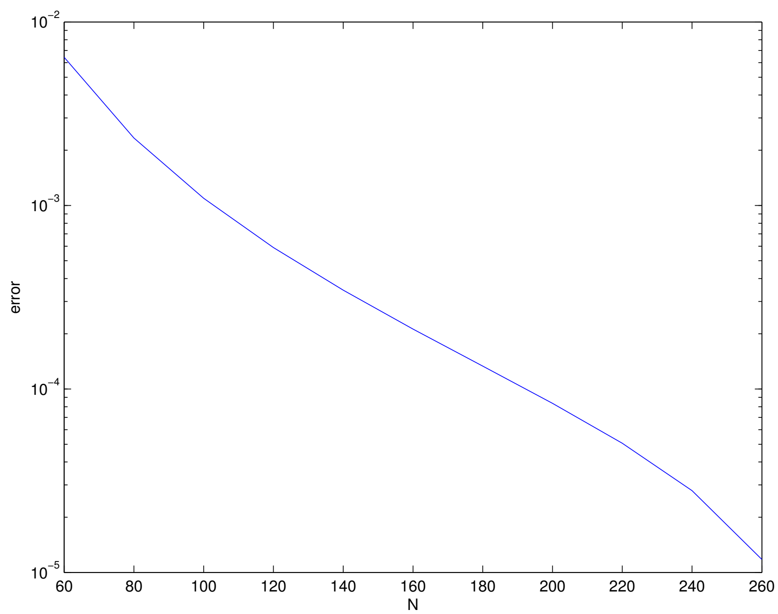
Note how u goes to 0 near $\partial\Omega$.

(b) Code: scathelmconv.m

The real part of the value of u at $x = (-2, 0)$ is about 1.2762.

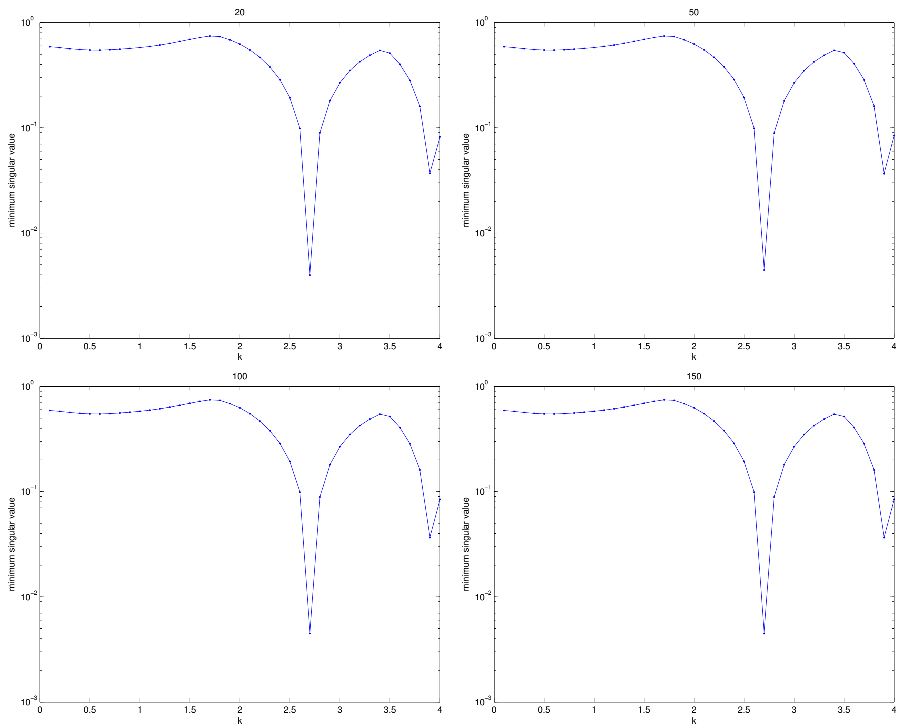


u at $x = (-2, 0)$ vs. N nodes.



Convergence of u at $x = (-2, 0)$ to $\text{Re}(u) = 1.2762$. Note how u is at 4 digits of accuracy at about 200 nodes.

3. (a) Code: kereval.m



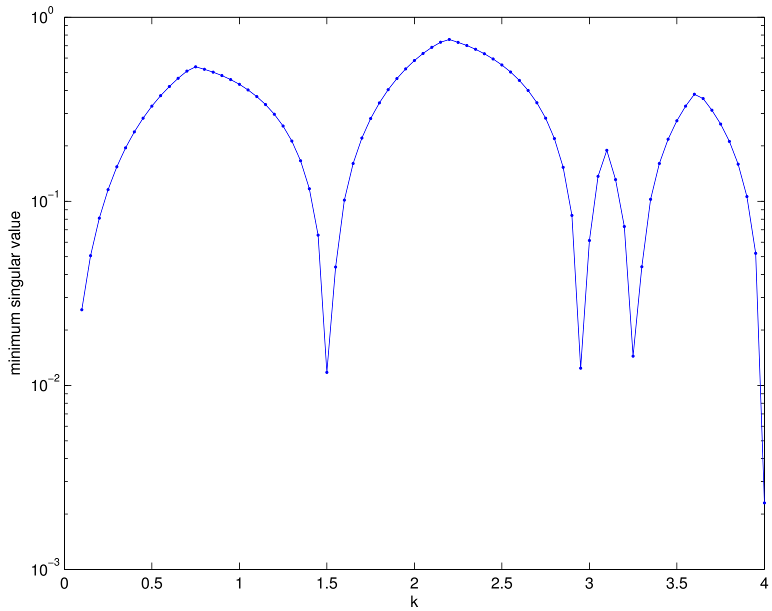
Dependence of k with minimum singular value on number of nodes:
Top: m=20, m=50 nodes; Bottom: m=100, m=150 nodes

The above plots seem to indicate that the k with the minimum singular value doesn't depend on the number of nodes used. Using the built-in MATLAB function fminsearch, we find that the smallest SVD value for 50 nodes occurs at $k \approx 2.7047$. The SVD value is approximately1.141×10^{-5} for this k.

Repeating the use of fminsearch for m=20 and m=30 nodes returned very close k values. The Singular Value differed for m=20 (about 10 times larger), but the Singular value for m=30 was on the same order.

(b) For this k value, k^2 is a Dirchlet eigenvalue of Ω, so the actual physical solution can blow up near this k, since for some u, $(\Delta + k^2)u = 0$ and $u|_{\partial\Omega} = 0$ so we can add this u multiplied by any constant to a particular solution v that matches the boundary data and also solves the Helmholtz equation, and still match the boundary data. Thus, the effect is not just particular to the BIE, but the problem itself.

(c) Code in second half of kereval.m



Dependence of minimum singular value on k.

Examining the plot, we see that there are potential minima at $k \approx 1.5, 3, 3.25$, and 4. Using fminsearch again, we find that the k with the minimum singular value is $k \approx 1.5107$ with a minimum singular value of 6.4065×10^{-5}.

(d) For this k, k^2 is a Neumann eigenvalue of Ω. This means that there exists some u that satisfies $(\Delta + k^2)u = 0$ and $u_n|_{\partial\Omega} = 0$. However, we are solving a Dirichlet problem, so and since $u|_{\partial\Omega}$ is not necessarily 0, we can not add this u multiplied by any constant to a particular solution v that satisfies the boundary data and the Helmholtz equation, and still have a function that satisfies the boundary data. Thus, this singular value is a BIE effect, as the Neumann eigenvalues should not affect the Dirichlet solution.