I am interested in the spatial statistics of random sums of plane waves, a special case of a Gaussian random field with a certain correlation function. This is a useful model for high-frequency eigenfunctions of the Laplacian (i.e. 'modes of a drum') in a domain (manifold) with chaotic geodesic flow. I am a numerical analyst and applied mathematician; my most relevant work is on accurate and efficient solution of the above-mentioned eigenfunctions, and other wave eigenvalue and scattering problems, in planar domains. I also have a background in quantum physics, and have used such numerical tools to test conjectures about short-wavelength asymptotics of eigenfunctions (quantum chaos).

To be more specific, one definition of a 'random sum of plane waves' of unit wavenumber is, for $x \in \mathbb{R}^2$,

$$u(x) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \mathrm{Re} \sum_{n=1}^{N} a_n e^{i(\cos\theta_n, \sin\theta_n)\cdot x}$$

where a_n are iid complex Gaussian random variables of unit variance, and the wavevector directions are $\theta_n = \pi n/N$. Alternatively one may describe u as Gaussian white noise restricted to the unit circle, followed by an inverse Fourier transform. The autocorrelation function is simple to compute, being a J_0 Bessel function of radial distance. I wish to understand spatial correlations over large distances in more subtle features such as an extreme level set. This is motivated by visual observations (see pictures below) that samples from the random plane wave distribution look 'stringy', i.e. high-value regions seem spatially correlated in a non-trivial way. We (and other researchers such as M. V. Berry) suspect that there is a fractal dimension to such level sets, at least within some range of length-scales. However, what exactly the eye is perceiving here does not yet have a mathematical description of which I am aware.

My main collaborative goals are to come up with such a description, measure statistical properties by numerical experiments, and prove statements about these random waves.

I have set up a webpage with some motivating and quite pretty pictures here: `http://math.dartmouth.edu/~ahb/rpws/` You may also download my fast Matlab code which uses a non-uniform FFT to compute samples from this distribution.

Some of my relevant publications are:

A. H. Barnett and T. Betcke, "Quantum mushroom billiards," *CHAOS* **17**, 043123 (2007) (13 pages) `nlin.CD/0611059`

A. H. Barnett, "Asymptotic rate of quantum ergodicity in chaotic Euclidean billiards", *Comm. Pure Appl. Math.* **59**, 1457–1488 (2006), `math-ph/0512030`

Other relevant publications include:

M. V. Berry, "Regular and irregular semiclassical wavefunctions," *J. Phys. A*, **10**(12), 2083–2091 (1977).

P. O'Connor, J. Gehlen, and E. J. Heller, "Properties of random superpositions of plane waves," *Phys. Rev. Lett.* **58**, 1296–1299 (1987)

S. Zelditch, "A random matrix model for quantum mixing," *Internat. Math. Res. Notices*, **3**, 115–137 (1996).

Cover image for *Notices of the American Mathematical Society*, January 2008, and Z. Rudnick's review article on quantum chaos in that issue.

There are also recent works by Toth, Wigman, Zelditch, etc, on nodal lines that are relevant.