BIEST: A Fast High-Order BIE Solver for Computing Stepped Pressure Equilibria in Stellarators

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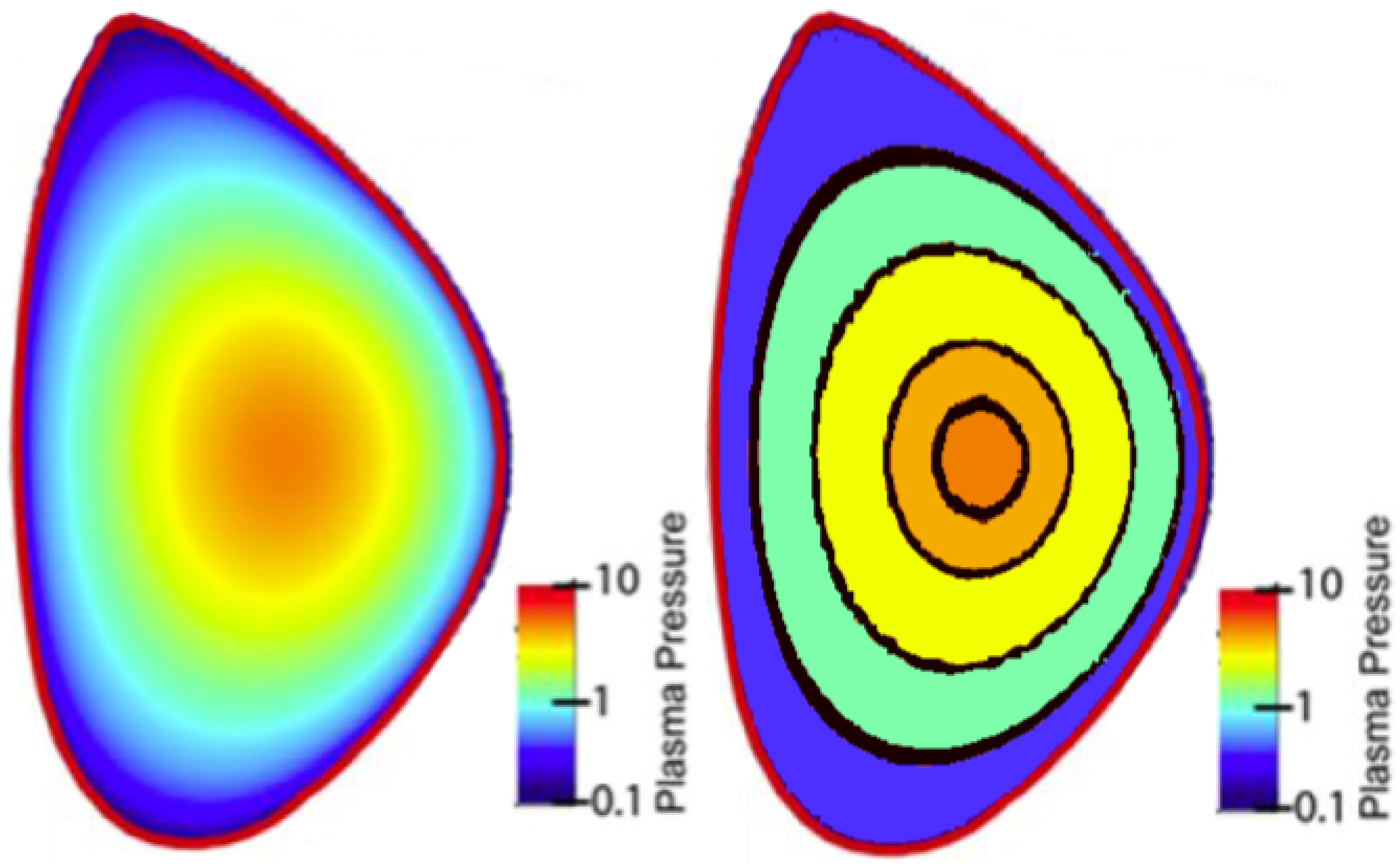
MHD Equilibrium Problem



$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \iff \nabla \times \mathbf{B} = \lambda \mathbf{B}$$

$$\mathbf{B} \cdot \mathbf{n} = 0 \qquad \text{(on flux surface)}$$

$$\langle p + \mathbf{B}^2/2 \rangle = 0 \qquad \text{(force balance)}$$

Relation to Time Harmonic Maxwell's Equations

$$\mathbf{H} = \mathbf{B} \qquad \text{and} \qquad \mathbf{E} = i\mathbf{B}$$

$$\nabla \times \mathbf{H} = -ik\mathbf{E}, \quad \nabla \times \mathbf{E} = ik\mathbf{H}$$

Generalized Debye representation
for time harmonic Maxwell's equations
C. Epstein, L. Greengard, M. O'Neil

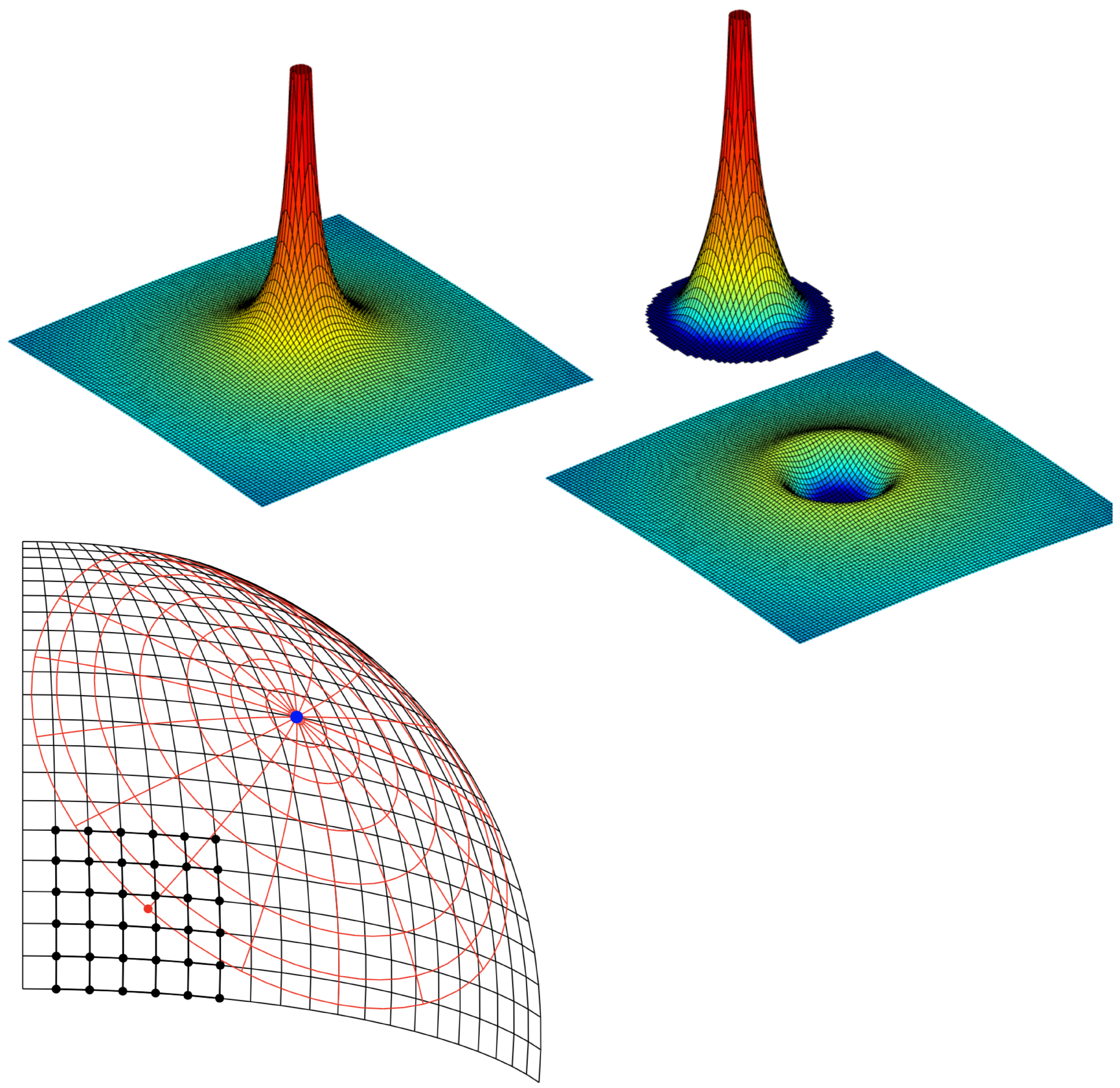
Boundary Integral Solver

- **Unknowns only on boundary.**
- **Well conditioned linear system.**
- **Fast and parallelizable.**
- **High order accurate.**

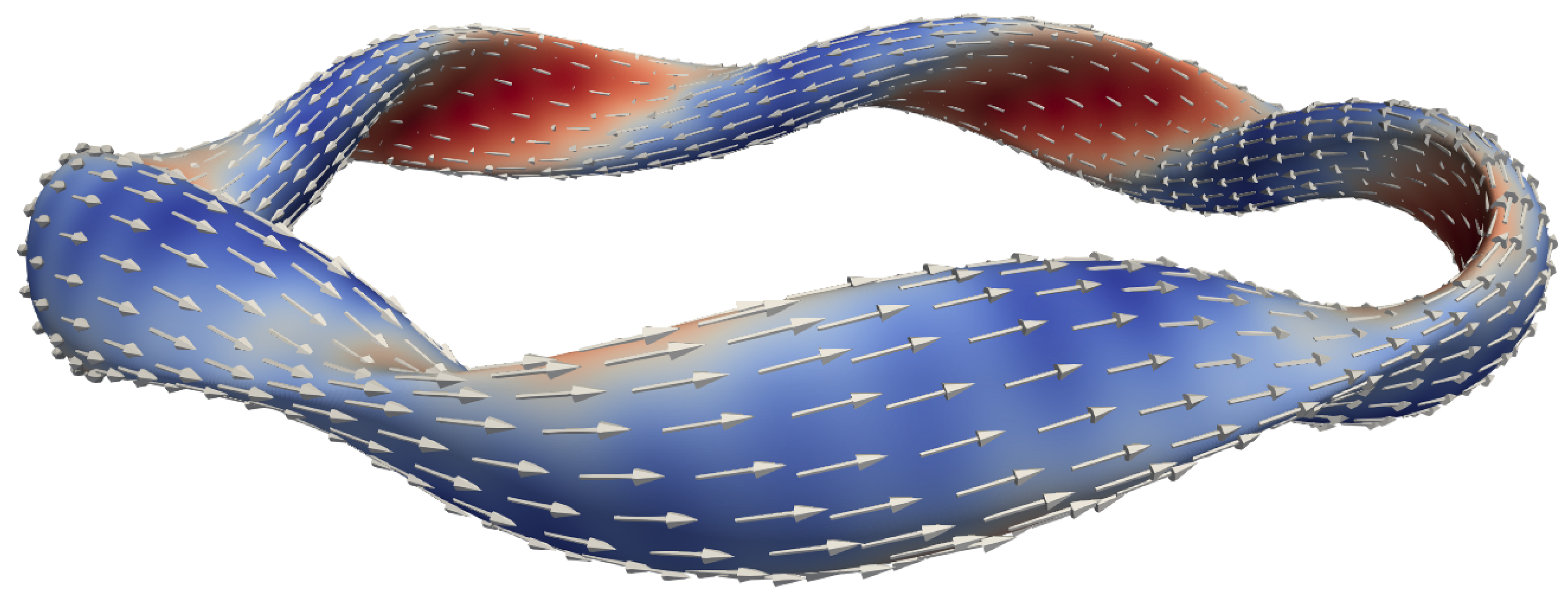
High-Order Singular Quadratures

$$\phi(\mathbf{x}) = \int_S K(\mathbf{x} - \mathbf{x}') f(\mathbf{x}') da'$$

$$K(\mathbf{r}) = \frac{1}{4\pi|\mathbf{r}|}$$

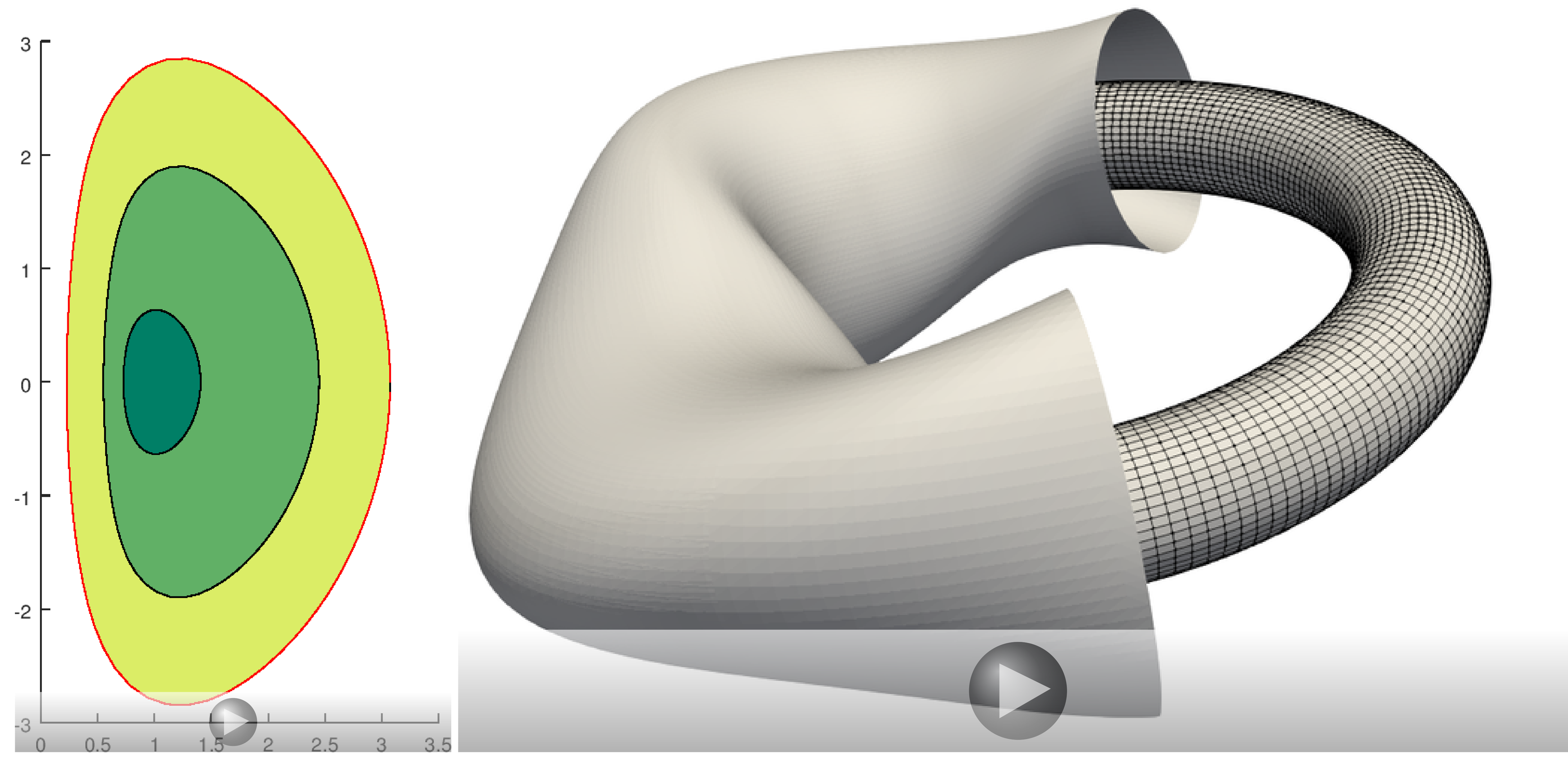


Numerical Results



BIEST			SPEC	
N	$\lvert e \rvert_\infty$	T_{solve}	$\lvert e \rvert_\infty$	T_{solve}
8.8E+3	6E-2	4.4	6E-2	38
3.5E+4	3E-3	61.6	2E-3	541
1.9E+5	9E-6	1991.0	-	-
7.7E+5	2E-9	40646.2	-	-

Equilibrium Calculation



- Move boundary in normal direction by distance proportional to pressure jump.
- Ongoing work: compute true gradient using adjoint formulation.

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