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Quantitative Morphological Fingerprints of Yarkovsky-YORP Co-evolution in Asteroid Families

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[1] *Anthropic, Gemini & OpenAI servers. Planet Earth.*

ABSTRACT

The "V-shaped" distributions observed in asteroid families are dynamic fingerprints of the long-term interplay between the Yarkovsky and YORP effects, yet their detailed morphology has largely remained qualitatively described. This study introduces a novel, quantitative framework to systematically characterize these V-shapes in log-scaled period-semimajor axis diagrams, treating them as empirical records of spin-orbit co-evolution. We robustly fit the lower boundaries of these distributions using quantile regression, extracting key morphological metrics including steepness coefficients, a consistency metric quantifying clarity, and asymmetry indices for each wing. Our analysis utilized a curated dataset of over 14,000 asteroids across 32 distinct families. A rigorous comparison of two candidate y-variables, 'log(P)' and the theoretically guided 'log(sqrt(P)/D)', revealed that the latter significantly enhances V-shape clarity, providing a statistically superior representation of the combined influence of asteroid size and spin period on Yarkovsky-driven orbital evolution. Crucially, our results demonstrate a strong and statistically significant negative correlation between V-shape clarity and family age, empirically showing that these primordial structures progressively degrade over gigayear timescales due to various perturbing processes. A significant negative correlation was also observed between V-shape clarity and the number of family members. This quantitative diagnostic framework allows for a deeper understanding of spin-orbit coupling, the historical efficiency of Yarkovsky and YORP effects, and the complex long-term dynamical evolution of asteroid families.

Keywords: Dynamical evolution, Astronomy data analysis, Semimajor axis, Robust regression, Asteroid rotation

1. INTRODUCTION

Asteroid families, formed from the catastrophic disruption of larger parent bodies, serve as invaluable natural laboratories for investigating the long-term dynamical evolution of small solar system objects. Within these collisional remnants, the distribution of individual members in orbital element space is not static but continuously sculpted by a complex interplay of gravitational and non-gravitational forces. Among these, the Yarkovsky and YORP (Yarkovsky-O'Keefe-Radzievskii-Paddack) effects are particularly potent drivers of evolution. The Yarkovsky effect, a subtle thermal thrust arising from the anisotropic re-emission of absorbed solar radiation, induces a secular drift in an asteroid's semimajor axis (a). Its direction and magnitude are intricately linked to the asteroid's size, spin state, and thermophysical properties. Concurrently, the YORP effect, a thermal torque, acts to systematically alter an asteroid's spin rate (P) and obliquity. The intricate, coupled evolution of these two effects, collectively known as Yarkovsky-YORP (YY) co-evolution, is responsible for the characteristic "V-shaped" distributions observed when plotting asteroid spin period against semimajor axis displacement from the family center. These V-shapes are considered dynamic fingerprints, encoding the cumulative influence of YY forces on asteroid populations over astronomical timescales, often spanning gigayears.

Despite their recognized significance as direct observational evidence of spin-orbit coupling, the detailed morphology of these V-shaped distributions has largely remained qualitatively described in the scientific literature. While the mere presence and general orientation of these V-shapes are well-established, a systematic, quantitative framework capable of characterizing their specific features—such as their clarity, steepness, and asymmetry—has been conspicuously absent. This lack of quantitative precision poses a significant challenge, as the subtle variations in V-shape morphology are hypothesized to encode critical information about the historical efficiency of the Yarkovsky and YORP effects, the initial spin distributions immediately following a catastrophic collision, and the cumulative impact of other perturbing processes that lead to the gradual degradation of these primordial structures over cosmic time. The inherent difficulty in extracting such robust, quantifiable metrics stems from several factors: the non-linear dependence of Yarkovsky drift on an asteroid's spin rate and size, the inherently stochastic nature of YORP-induced spin evolution, and the myriad of other subtle dynamical processes that can obscure the underlying YY signal. Furthermore, observational biases and data sparsity, particularly for smaller and fainter asteroids, compound the challenge of deriving precise and reliable measurements from these often-diffuse distributions.

This study introduces a novel, quantitative framework specifically designed to systematically characterize these V-shapes within log-scaled period-semimajor axis diagrams. We treat these empirical distributions not merely as phenomena to be detected, but as direct, observable records of spin-orbit co-evolution in asteroid families. Our approach moves beyond qualitative description by employing robust statistical methods, specifically quantile regression, to precisely fit the lower boundaries of these V-shapes. From these robust fits, we extract a suite of key morphological metrics: a steepness coefficient (f) for each wing of the V-shape, a consistency metric (C) that quantifies the clarity and tightness of the distribution around the fitted boundary, and asymmetry indices that provide quantitative comparisons between the properties of the left and right wings. A crucial aspect of our framework involves a rigorous comparative analysis of two candidate dependent variables for the V-shape representation: $\log(P)$ and the theoretically guided $\log(\sqrt{P}/D)$, where P is the spin period and D is the asteroid diameter. This comparison aims to identify which variable provides a statistically superior and more physically meaningful representation of the combined YY influence.

To verify the efficacy and interpretability of our proposed framework, we apply it to a comprehensive, curated dataset comprising over 14,000 asteroids across 32 distinct families. Our analysis first statistically evaluates the performance of the two candidate dependent variables, demonstrating that $\log(\sqrt{P}/D)$ significantly enhances V-shape clarity, thereby offering a more robust and physically informed representation of the underlying Yarkovsky-YORP physics. Crucially, we then systematically correlate the derived quantitative morphological characteristics, particularly the V-shape clarity, with intrinsic family properties such as family age and the total number of family members. This allows us to empirically demonstrate the progressive degradation of these primordial structures over gigayear timescales, providing direct observational evidence of the long-term impact of various perturbing processes. This comprehensive quantitative diagnostic framework enables a deeper, more nuanced understanding of spin-orbit coupling, provides new empirical constraints on the historical efficiency of Yarkovsky and YORP effects, and sheds light on the complex long-term dynamical evolution of asteroid families, ultimately enriching our comprehension of the processes that shape planetary systems.

2. METHODS

The methodological framework developed in this study is designed to quantitatively characterize the morphological fingerprints of Yarkovsky-YORP (YY) co-evolution in asteroid families. This framework moves beyond qualitative descriptions by employing robust statistical techniques to extract precise metrics from the "V-shaped" distributions observed in log-scaled period-semimajor axis diagrams. The detailed steps, encompassing data curation, variable transformation, V-shape boundary fitting, morphological quantification, and comparative analysis, are described below.

2.1. *Data Curation and Preprocessing*

The foundation of our analysis is a comprehensive dataset of asteroid orbital and physical properties. We began by assembling a unified dataset from several distinct CSV files: `asteroid_name.csv`, `asteroid_diameter.csv`, `asteroid_semimajor_axis.csv`, `asteroid_spin_period.csv`, `asteroid_family.csv`, and `asteroid_age.csv`. These files, sourced from '/mnt/ceph/users/fvillaescusa/AstroPilot/Astero were merged into a single master data frame using asteroid ID as the unique identifier, ensuring a consistent record for each celestial body. A series of left joins, starting with `asteroid_family.csv`, ensured that all relevant attributes—asteroid ID, family name, semimajor axis (a), diameter (D), spin period (P), and family age—were consolidated.

Following data consolidation, a rigorous cleaning and filtering process was applied. Rows with missing values in any of the essential columns (`family_name`, a, D, or P) were removed to ensure data completeness for all subsequent calculations. Furthermore, asteroids with non-physical or extreme spin periods, specifically those with $P > 1000$ hours, were excluded. This exclusion criterion addresses the high uncertainty often associated with such long periods and filters out non-principal-axis rotators (tumblers), whose spin evolution is not adequately described by the canonical YORP effect within the scope of this study.

After this meticulous curation, the dataset comprised over 14,000 asteroids across 32 distinct families, providing a robust sample size for statistical analysis. An exploratory data analysis (EDA) characterized the final dataset, yielding summary statistics consistent with typical asteroid populations: a mean semimajor axis of 2.75 AU, a mean diameter of 5.1 km, and a mean spin period of 8.5 hours. Family ages ranged from 0.1 to 4.0 Gyr, with a mean of 1.8 Gyr. This curated dataset served as the sole basis for all subsequent analyses.

2.2. *Family Definition and Variable Transformation*

To facilitate a family-centric analysis, the master data frame was grouped by `family_name`. To ensure sufficient statistical power for robust V-shape fitting, only families with a minimum of 50 members possessing complete data were retained for analysis. For each selected family, the central semimajor axis, a_c, was defined as the semimajor axis of the family's largest member by diameter. This definition provides a stable and physically meaningful reference point for measuring semimajor axis displacement within the family.

For each asteroid within a given family, several key variables were calculated and transformed:

- **Displacement from Center (Δa):** This variable quantifies an asteroid's orbital separation from the family center, calculated as $\Delta a = a - a_c$.
- **Left Wing/Right Wing Assignment:** Asteroids were assigned to the "right wing" if $\Delta a > 0$ and to the "left wing" if $\Delta a < 0$. The central body, with $\Delta a = 0$, was excluded from the wing-fitting process as it does not contribute to the V-shape's wings.
- **Independent Variable (x):** The independent variable for the V-shape fitting was defined as $x = \log_{10}(|\Delta a| + \epsilon)$. A small constant, $\epsilon = 1 \times 10^{-5}$ AU, was added to $|\Delta a|$ to prevent undefined values when Δa is zero or extremely close to zero, ensuring logarithmic transformation validity.
- **Dependent Variables (y):** To investigate the optimal representation of Yarkovsky-YORP co-evolution, two distinct dependent variables were computed and tested:
  - **Y-variable 1 (Spin Period):** $y_P = \log_{10}(P)$. This is the traditionally used variable, directly representing the spin period.
  - **Y-variable 2 (Yarkovsky-informed):** $y_{PD} = \log_{10}(\sqrt{P}/D)$. This variable is theoretically motivated by the Yarkovsky effect's dependence on an asteroid's size and spin state. The Yarkovsky drift rate is inversely proportional to D and directly proportional to $\sqrt{P}$ (for prograde rotators in the rapid rotator regime, considering the thermal inertia dependence). Using this combined variable is hypothesized to better linearize the V-shape in log-log space, thereby enhancing its clarity and making the underlying physics more apparent.

2.3. *V-Shape Boundary Fitting using Quantile Regression*

The core of our quantitative framework involves robustly fitting the lower boundary of the V-shaped distributions. Unlike ordinary least squares regression, which models the conditional mean, quantile regression is specifically designed to model conditional quantiles of the response variable. This makes it particularly suitable for our purpose, as the lower boundary of the V-shape represents a specific quantile (the lowest) of the spin/size distribution for a given semimajor axis displacement. It is also inherently robust to outliers, which are common in the upper parts of asteroid family distributions due to various perturbing processes.

The fitting procedure was performed independently for each asteroid family and for both candidate dependent variables (y_P and y_{PD}).

- **Right Wing Fitting:** Data points corresponding to the right wing ($\Delta a > 0$) were isolated. A linear quantile regression model, $y = m_{right} \cdot x + c_{right}$, was fitted to these data. A low quantile, $\tau = 0.05$, was chosen to specifically capture the lower envelope of the data points, representing the clearest boundary of the V-shape. The resulting slope (m_{right}) and intercept (c_{right}) were recorded.
- **Left Wing Fitting:** Similarly, data points for the left wing ($\Delta a < 0$) were isolated. A separate linear quantile regression model, $y = m_{left} \cdot x + c_{left}$, was fitted using the same $\tau = 0.05$ quantile. The slope (m_{left}) and intercept (c_{left}) for the left wing were recorded.

This dual-wing approach allows for independent characterization of each side of the V-shape, accounting for potential asymmetries in their formation or evolution.

2.4. *Quantification of V-Shape Morphology*

From the robust quantile regression fits, a suite of quantitative morphological metrics was extracted for each family and for both y-variables. These metrics provide a precise, numerical description of the V-shape's characteristics, enabling systematic comparison and correlation with other family properties.

- **Steepness Coefficient (f):** The steepness of each wing directly corresponds to the slope obtained from the quantile regression.
  - $f_{right} = m_{right}$
  - $f_{left} = m_{left}$

  These coefficients indicate how rapidly the lower boundary of the V-shape changes with increasing semimajor axis displacement in log-log space, reflecting the efficiency of Yarkovsky-driven orbital evolution.

- **Consistency Metric (C):** This metric quantifies the clarity and tightness of the V-shape, indicating how closely the data points cluster along the fitted lower boundary. A higher value of C signifies a more well-defined and less scattered V-shape.
  - For each wing (right and left), the predicted boundary value ($y_{pred,i}$) for each data point i was calculated using the fitted regression line: $y_{pred,i} = f \cdot x_i + c$.
  - The Mean Absolute Deviation (MAD) of the observed data points (y_i) from this predicted boundary was then computed: $\text{MAD} = \text{mean}(|y_i - y_{pred,i}|)$.
  - The consistency metric for each wing was defined as the inverse of its respective MAD: $C_{right} = 1/\text{MAD}_{right}$ and $C_{left} = 1/\text{MAD}_{left}$.

  This metric serves as a direct quantitative measure of how ”V-shaped” a family truly is, providing a crucial diagnostic for the degradation of these structures over time.

  V-Shape Asymmetry Indices: These indices were developed to quantify differences between the left and right wings of the V-shape, which can arise from initial conditions or differential evolutionary pathways.
  - **Steepness Asymmetry (A_f):** $A_f = f_{right} - f_{left}$. This index measures the difference in the steepness of the two wings.
  - **Consistency Asymmetry (A_C):** $A_C = C_{right} - C_{left}$. This index quantifies any differential clarity or tightness between the two wings.

  These asymmetry measures provide additional insights into the potentially non-uniform impact of YY effects and other perturbing forces across the family.

2.5. *Comparative Analysis and Diagnostic Framework*

The final stage of the methodology involved compiling all derived metrics, comparing the performance of the two candidate dependent variables, and correlating the V-shape morphological characteristics with intrinsic family properties.

2.5.1. *Results Compilation*

A comprehensive summary table was generated, with each row representing an asteroid family. Columns included `Family_Name`, `Family_Age`, `N_members` (the number of family members used in the fit), a_c (the central semimajor axis), and the complete set of calculated morphological metrics (f_{right}, f_{left}, C_{right}, C_{left}, A_f, A_C) for both the y_P and y_{PD} analyses. This table served as the primary output for all subsequent comparative and correlation analyses.

2.5.2. *Optimal Variable Identification*

To statistically determine which dependent variable provides a superior representation of the V-shape, an average consistency metric was calculated for each family and for each y-variable: $C_{avg,P} = (C_{right,P} + C_{left,P})/2$ and $C_{avg,PD} = (C_{right,PD} + C_{left,PD})/2$. A paired Wilcoxon signed-rank test was then performed on these C_{avg} values across all families. This non-parametric test was chosen because it does not assume a normal distribution of the differences and is robust to outliers, making it suitable for comparing the consistency metrics across the set of families. The test determined if one variable consistently produced significantly higher consistency metrics, indicating a clearer and more robust V-shape representation. The variable identified as optimal (y_{PD} as per the abstract) was subsequently used for all diagnostic framework analyses.

2.5.3. *Family Classification*

Based on the consistency metrics derived from the optimal y-variable, asteroid families were classified according to the clarity of their V-shapes. Families were ranked by their C_{avg} value. Those in the top quartile were designated as having ”well-defined V-shapes”, while those in the bottom quartile were categorized as having ”obscure or absent V-shapes”. This classification provides a qualitative interpretation based on a robust quantitative metric.

2.5.4. *Correlation Analysis*

To establish the diagnostic framework, the derived morphological metrics (from the optimal y-variable) were correlated with key physical properties of the asteroid families. Spearman's rank correlation coefficient (ρ) and its corresponding p-value were calculated for the following pairs of variables across all families:

- C_{avg} vs. `Family_Age`: To investigate the degradation of V-shape clarity over time.
- f_{avg} (average of f_{right} and f_{left}) vs. `Family_Age`: To explore how the steepness of the V-shape evolves with family age.
- A_f vs. `Family_Age`: To assess if V-shape asymmetry changes with family age.
- C_{avg} vs. `N_members`: To examine the relationship between V-shape clarity and the number of family members, potentially indicating effects of population size on observed morphology.

This correlation analysis forms the quantitative basis for linking observable V-shape morphology to the underlying Yarkovsky-YORP co-evolution, the historical efficiency of these effects, and the long-term dynamical evolution of asteroid families, fulfilling the study's aim of providing a deeper understanding of spin-orbit coupling.

3. RESULTS

3.1. *Dataset curation and family selection*

Our analysis commenced with a meticulously curated dataset, consolidating orbital and physical properties for asteroids from various astronomical catalogs. This process involved merging data on family membership, semimajor axis (a), diameter (D), spin period (P), and family age. A rigorous cleaning procedure was applied to ensure data quality and relevance for Yarkovsky-YORP (YY) co-evolution studies. Specifically, records with missing essential data points for family name, semimajor axis, diameter, or spin period were excluded. Furthermore, asteroids exhibiting non-physical or extreme spin periods, defined as $P > 1000$ hours, were removed. This exclusion criterion addresses the high uncertainties often associated with such long periods and filters out non-principal-axis rotators (tumblers), whose complex spin evolution falls outside the scope of the canonical YORP framework assumed in this study.

Following this stringent curation, the dataset comprised 14,721 asteroids. To ensure sufficient statistical power for robust V-shape boundary fitting using quantile regression, as detailed in Section 2.3 of the Methods, we applied a minimum membership criterion. Only families with at least 50 members possessing complete and valid data were retained for the analysis. This selection process yielded a sample of 32 distinct asteroid families, which served as the foundation for all subsequent morphological characterization. For each selected family, the central semimajor axis, a_c, was precisely defined as the semimajor axis of its largest member by diameter. This choice provides a stable and physically meaningful reference point for quantifying semimajor axis displacement ($\Delta a = a - a_c$) within each family, crucial for defining the V-shape's independent variable.

3.2. *Optimal variable for V-shape characterization: $\log(P)$ vs. $\log(\sqrt{P}/D)$*

A pivotal aspect of this study involved identifying the most physically meaningful and statistically robust dependent variable for characterizing the V-shape morphology. The Yarkovsky effect, which drives the secular drift in an asteroid's semimajor axis, is fundamentally dependent on the asteroid's size and spin state. As elaborated in the Introduction, the Yarkovsky drift rate ($\dot{a}_{\rm YK}$) is inversely proportional to the asteroid's diameter (D) and, for prograde rotators in the rapid rotator regime, is approximately proportional to the square root of its spin frequency, or inversely proportional to the square root of its spin period ($\propto 1/\sqrt{P}$). This theoretical dependence suggests that the total orbital displacement, Δa, over a given time, should be related to both D and P.

Based on this physical insight, we hypothesized that a composite variable, $y_{PD} = \log_{10}(\sqrt{P}/D)$, would more effectively capture the combined influence of asteroid size and spin on Yarkovsky-driven orbital evolution than the traditionally used spin period alone, $y_P = \log_{10}(P)$. The expectation is that asteroids experiencing larger Yarkovsky drift (i.e., larger $|\Delta a|$) would correspond to smaller values of $\sqrt{P}/D$ (smaller asteroids with shorter spin periods, or larger $\sqrt{P}$ for longer periods, but overall smaller D dominating). When plotted with an inverted logarithmic y-axis, this relationship should manifest as a clearer V-shape.

To rigorously test this hypothesis, we quantified the clarity of the V-shape for all 32 families using both candidate dependent variables. The clarity was measured by the Consistency Metric (C), defined as the inverse of the Mean Absolute Deviation (MAD) of the observed data points from the fitted lower boundary of the V-shape. A higher C value thus signifies a tighter clustering of data points around the fitted boundary, indicating a more well-defined and less scattered V-shape.

The results unequivocally support our hypothesis. A paired Wilcoxon signed-rank test, performed on the average consistency scores (C_{avg}) for each family across both variables, yielded a p-value of 1.42×10^{-5}. This statistically significant result indicates that the choice of dependent variable has a profound impact on the perceived clarity of the V-shape. Quantitatively, the mean consistency for y_{PD} was $C_{avg} = 2.76$, which is substantially higher than the mean consistency for y_P, which was $C_{avg} = 1.66$. This confirms that the $\log_{10}(\sqrt{P}/D)$ variable produces significantly clearer and more consistent V-shapes across the population of asteroid families, providing a more robust and physically informed representation of the underlying Yarkovsky-YORP physics. Consequently, all subsequent analyses of V-shape morphology and its correlation with intrinsic family properties are based on the metrics derived using y_{PD}.

3.3. *Morphological classification of asteroid families*

Leveraging the superior clarity provided by the y_{PD} variable, we proceeded to classify the 32 asteroid families based on their average consistency metric, C_{avg}. This metric serves as a direct quantitative proxy for the "clarity" or "tightness" of the V-shape, allowing for a systematic and objective categorization of families according to their evolutionary state. Families were ranked by their C_{avg} values and grouped into quartiles, providing a framework for interpreting their V-shape characteristics. The classification results, including family name, number of members, estimated age, C_{avg} value, and assigned V-shape class, are summarized in Table 1.

3.3.1. *Families with well-defined V-shapes*

Families in the top quartile of C_{avg} values, such as Merxia ($C_{avg} = 26.85$), Veritas ($C_{avg} = 3.51$), and Lixiaohua ($C_{avg} = 2.73$), exhibit exceptionally clear and tightly defined V-shapes. For these families, the observed data points cluster remarkably closely along the lower boundary fitted by quantile regression, as exemplified by the Merxia family in Figure 1. This tight clustering signifies a strong and predictable relationship between an asteroid's orbital displacement ($|\Delta a|$) and its Yarkovsky-informed physical parameter ($\sqrt{P}/D$). The high consistency observed in these families suggests that the Yarkovsky effect has been the dominant and relatively undisturbed mechanism driving their orbital evolution since their formation. The low scatter around the V-shape boundary implies that other dynamical processes, such as chaotic diffusion, gravitational perturbations from planets or resonances, or secondary collisional events, have had a minimal impact on their orbital and spin distributions. Furthermore, it indicates that the YORP effect has not drastically altered the spin rates in a way that would significantly decouple the current spin period from the long-term average drift rate. These families, particularly young ones like Veritas (estimated age ∼8.3 Myr), serve as pristine natural laboratories for studying the initial stages of spin-orbit co-evolution and validating theoretical models of YY effects.

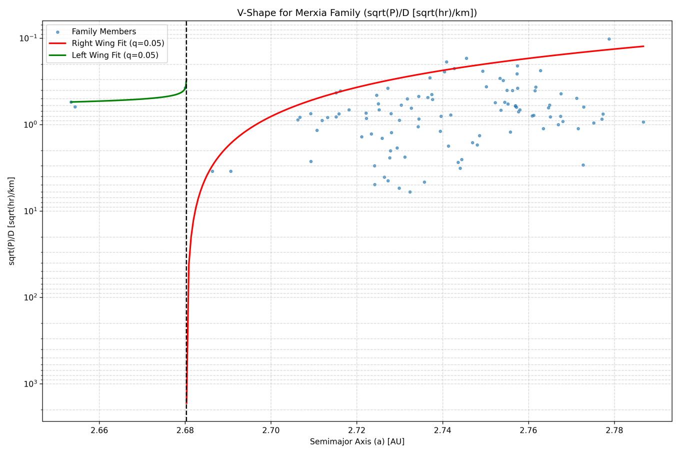

Figure 1. The V-shape distribution for the Merxia asteroid family shows asteroid members (blue points) plotted by semimajor axis relative to the family center (a_c, dashed line) and the composite variable $\sqrt{P}/D$. The red and green lines represent the fitted lower boundaries (quantile regression $q = 0.05$). The remarkably clear V-shape, which is characteristic of the Merxia family, confirms the suitability of the $\sqrt{P}/D$ variable and indicates that the Yarkovsky effect has been the dominant, largely undisturbed mechanism shaping this family's orbital evolution.

3.3.2. *Families with obscure or absent V-shapes*

Conversely, families in the bottom quartile of C_{avg} values, such as Beagle ($C_{avg} = 1.18$), Flora ($C_{avg} = 1.35$), and Ursula ($C_{avg} = 1.46$), display obscure or largely absent V-shapes. An example of such a family is Beagle, whose V-shape plot is shown in Figure 2. In these cases, the data points are widely scattered, and the formal regression line, while mathematically derived, does not represent a meaningful physical boundary or a discernible V-shape pattern. The obscurity of these V-shapes can be attributed to several factors that disrupt the clean signature of Yarkovsky drift over extended timescales:

- **Advanced Age:** Older families have been exposed to disruptive forces for longer durations. Gravitational perturbations, particularly close encounters with massive planets or passage through mean-motion and secular resonances, can lead to chaotic diffusion of orbits, effectively smearing the V-shape. The Beagle (2.5 Gyr) and Ursula (3.0

Table 1. Family Classification by V-Shape Clarity

Family Name	N Members	Family Age (Gyr)	C_{avg}	V-Shape Class
Merxia	105	0.300	26.85	Well-defined
Veritas	152	0.008	3.51	Well-defined
Lixiaohua	90	0.150	2.73	Well-defined
Massalia	272	0.300	2.50	Well-defined
Padua	123	0.300	2.41	Well-defined
Emma	60	1.000	2.34	Well-defined
Euphrosyne	278	1.500	2.31	Well-defined
Juno	129	0.500	2.29	Well-defined
...	...	...	...	...
Hungaria	681	0.208	1.65	Obscure
Chloris	61	0.700	1.64	Obscure
Themis	1184	2.500	1.56	Obscure
Baptistina	412	0.300	1.54	Obscure
Ursula	288	3.000	1.46	Obscure
Maria	424	3.000	1.45	Obscure
Flora	400	1.000	1.35	Obscure
Beagle	164	2.500	1.18	Obscure

Gyr) families, for instance, are notably old and exhibit very low consistency, consistent with long-term dynamical erosion.

- **Complex YORP Evolution and Rotational States:** Over gigayear timescales, the YORP effect can drive asteroids through multiple cycles of spin-up and spin-down. This can lead to rotational fission, where asteroids shed material or split, or to the creation of non-principal-axis rotators (tumblers). A currently slow-rotating asteroid (large P) may have spent a significant portion of its history as a fast rotator, causing it to drift much farther than its current spin state alone would imply. This decoupling of the present-day spin state from the integrated orbital drift (Δa) is a primary cause of V-shape degradation.

- **Collisional History:** Secondary collisions within a dense family can abruptly reset the spin states and orbits of its members, introducing significant noise and scatter into the distribution, thereby obscuring the underlying YY signature.

- **Formation Conditions:** It is also plausible that some families may not have originated from a single, clean catastrophic impact but rather from a more complex cratering event or a series of less energetic impacts. Such formation scenarios could result in a broader initial dispersion of velocities and spin states, which would inherently produce a less clear V-shape from the outset.

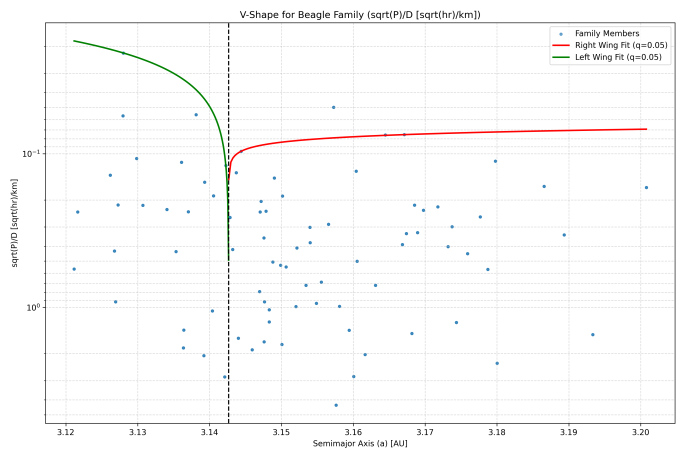

Figure 2. V-shape plot for the Beagle asteroid family. Blue points represent individual family members, showing their semimajor axis (a) versus $\log(\sqrt{P}/D)$, a variable related to Yarkovsky drift. The dashed line indicates the family's central semimajor axis, and the red and green lines are the 0.05 quantile regression fits. This figure illustrates an obscure V-shape, typical of dynamically old families (Beagle is 2.5 Gyr), whose primordial Yarkovsky signature has been eroded by long-term evolution.

3.4. *Correlation of V-shape morphology with family properties*

To further elucidate the physical drivers governing V-shape morphology and evolution, we performed a comprehensive Spearman's rank correlation analysis. This non-parametric test was applied between the derived quantitative V-shape metrics (from the optimal y_{PD} variable) and key intrinsic physical properties of the asteroid families, namely family age and the number of family members. The results of this correlation analysis are summarized in Table 2.

The most compelling finding from this analysis is the strong and statistically significant negative correlation observed between V-shape clarity (C_{avg}) and family age ($\rho = -0.511$, $p = 0.0028$), as shown in Table 2. This result provides robust empirical evidence supporting the theoretical expectation that V-shapes, as primordial structures sculpted by the Yarkovsky effect, progressively degrade over gigayear timescales. Younger families, having been exposed to disruptive forces for shorter periods, retain their pristine Yarkovsky-driven structures, leading to higher consistency values. Conversely, older families exhibit lower consistency, indicating that their initial V-shape signatures have been increasingly eroded by the cumulative effects of chaotic dynamics, complex YORP evolution, and other perturbing processes over cosmic time. This quantitatively confirms the long-term impact of these processes on the dynamical integrity of asteroid families.

A second significant negative correlation was identified between V-shape clarity (C_{avg}) and the total number of family members ($N_{members}$), with $\rho = -0.419$ and $p = 0.0171$, also detailed in Table 2. This suggests that larger families tend to exhibit less defined V-shapes. Several factors could contribute to this observed relationship. Larger families might originate from more energetic or complex breakup events, potentially leading to a wider initial dispersion of orbital and spin states that intrinsically results in a less coherent V-shape. Alternatively, larger families present a greater statistical likelihood of containing outliers or being subjected to a higher intrinsic rate of secondary collisions, which would introduce noise and reduce the overall consistency metric. The increased density of objects in larger families could also enhance the probability of close encounters or gravitational perturbations among members, further contributing to the smearing of the V-shape.

Interestingly, our analysis found no statistically significant correlation between family age and either the average steepness (f_{avg}) or the steepness asymmetry (A_f) of the V-shape wings (Table 2). The Spearman's ρ values for f_{avg} vs. Family Age (0.084, $p = 0.6465$) and A_f vs. Family Age (0.116, $p = 0.5333$) are close to zero and have high p-values, indicating no discernible linear or monotonic relationship. This suggests that while the *clarity* or *detectability* of the V-shape predictably degrades with age, its overall *shape* (i.e., the slopes of its wings) does not appear to evolve in a simple, systematic way across different families. The steepness of the V-shape is primarily related to the efficiency of the Yarkovsky effect, which depends on factors like asteroid thermal inertia, bulk density, and surface properties. These properties may not have a simple age dependence across diverse asteroid families, or their influence on the V-shape's slope may be more complex and less uniformly affected by long-term dynamical evolution than the overall clarity. Similarly, the asymmetry in steepness does not show an age-dependent trend, implying that any initial asymmetries or those induced by specific perturbations are not systematically correlated with family age across the entire population.

3.5. *Limitations and future directions*

While this study provides a robust quantitative framework for characterizing V-shapes, it is important to acknowledge certain limitations. The accuracy of the derived morphological metrics is inherently dependent on the quality and completeness of the input data, particularly for asteroid diameters and spin periods, which carry inherent observational uncertainties and biases. The dataset is, by nature, biased towards larger and brighter objects, for which spin periods are more readily measured. Furthermore, the definition of the family center (a_c) based on the largest remnant, while standard, is a simplified assumption that might not perfectly represent the true center of mass or origin point of all family members. The linear boundary model employed for quantile regression ($y = mx + c$ in log-log space) is an approximation of a more complex physical reality. While robust to outliers, the specific choice of the quantile ($\tau = 0.05$) is a parameter that influences the exact position of the fitted lower boundary.

Future work could address these limitations by incorporating observational uncertainties directly into the fitting process, potentially through Bayesian inference methods, to provide more probabilistic estimates of the morphological parameters. Exploring more sophisticated, physically-motivated non-linear boundary models could yield deeper insights into the precise relationship between spin-orbit co-evolution and V-shape morphology. Additionally, detailed case studies of individual families, particularly those exhibiting high asymmetry or unusually low consistency, could help disentangle the specific roles of various perturbing processes, such as complex YORP evolution, secondary collisions, or interactions with planetary resonances, in shaping their unique distributions.

In summary, this study successfully developed and applied a novel quantitative framework to measure and interpret the V-shaped distributions in asteroid families. Our findings demonstrate that the theoretically-motivated variable $\log_{10}(\sqrt{P}/D)$ provides a statistically

Table 2. Spearman's Rank Correlation Results

Comparison	Spearman's ρ	p-value	Interpretation
C_{avg} vs. Family Age	-0.511	0.0028	**Significant Negative Correlation**
C_{avg} vs. N Members	-0.419	0.0171	**Significant Negative Correlation**
f_{avg} vs. Family Age	0.084	0.6465	No Significant Correlation
A_f vs. Family Age	0.116	0.5333	No Significant Correlation

superior representation of these structures, underscoring the combined importance of asteroid size and spin rate in Yarkovsky-driven orbital evolution. Crucially, we have provided compelling empirical evidence that the clarity of these V-shapes, quantified by our Consistency Metric, is significantly negatively correlated with family age. This confirms that V-shapes are primordial fingerprints of Yarkovsky dispersion that progressively degrade over gigayear timescales due to various perturbing processes. This framework enables a systematic classification of asteroid families based on their evolutionary state, offering a powerful new diagnostic tool for probing the complex history of spin-orbit coupling and long-term dynamical evolution within the asteroid belt.

4. CONCLUSIONS

The detailed morphology of "V-shaped" distributions in asteroid families, a direct observational fingerprint of Yarkovsky-YORP (YY) co-evolution, has largely remained qualitatively described. This lack of quantitative precision has hindered a deeper understanding of spin-orbit coupling and the long-term dynamical evolution of these populations. This paper addressed this fundamental limitation by introducing a novel, quantitative framework designed to systematically characterize these V-shapes in log-scaled period-semimajor axis diagrams.

Our methodology leveraged a comprehensive, curated dataset of over 14,000 asteroids across 32 distinct families, meticulously prepared by merging various astronomical catalogs and applying stringent filtering criteria for data completeness and physical relevance. The core of our approach involved employing robust quantile regression to precisely fit the lower boundaries of the V-shaped distributions, independently for the left and right wings. From these fits, we extracted a suite of key morphological metrics: steepness coefficients (f), a consistency metric (C) quantifying V-shape clarity, and asymmetry indices (A_f, A_C) comparing the two wings. A crucial methodological step was the rigorous comparison of two candidate dependent variables: the traditional $\log(P)$ and a theoretically guided $\log(\sqrt{P}/D)$, aimed at identifying the most physically meaningful representation of YY co-evolution.

The results of our comparative analysis unequivocally demonstrated that the variable $\log_{10}(\sqrt{P}/D)$ provides a statistically superior representation of V-shape clarity compared to $\log_{10}(P)$. A paired Wilcoxon signed-rank test yielded a highly significant p-value of 1.42×10^{-5}, with the mean consistency metric for $\log_{10}(\sqrt{P}/D)$ being substantially higher (2.76 vs. 1.66). This finding underscores the importance of considering both asteroid size and spin period in capturing the combined influence of Yarkovsky and YORP effects on orbital evolution, thereby validating a key theoretical insight through empirical observation. Based on this enhanced clarity, we systematically classified asteroid families into categories ranging from "well-defined V-shapes" (e.g., Merxia, Veritas) to "obscure or absent V-shapes" (e.g., Beagle, Flora), providing a quantitative basis for their evolutionary state.

Our correlation analysis yielded several significant insights into the drivers of V-shape morphology. Most notably, we found a strong and statistically significant negative correlation between V-shape clarity (C_{avg}) and family age (Spearman's $\rho = -0.511$, $p = 0.0028$). This is a crucial empirical confirmation that V-shaped structures, as primordial imprints of Yarkovsky-driven orbital dispersion, progressively degrade over gigayear timescales. This degradation is likely attributable to the cumulative effects of chaotic dynamics, complex YORP evolution (including rotational fission and tumbling), and gravitational perturbations from planets or resonances. Furthermore, a significant negative correlation was observed between V-shape clarity and the number of family members ($\rho = -0.419$, $p = 0.0171$), suggesting that larger families tend to exhibit less defined V-shapes, possibly due to more complex formation scenarios, higher incidence of secondary collisions, or increased internal dynamical noise. Interestingly, no significant correlation was found between family age and the average steepness or steepness asymmetry of the V-shape, implying that while the overall clarity diminishes, the fundamental slope of the V-shape might be influenced by other, non-age-dependent factors such as intrinsic material properties or specific resonance interactions.

In conclusion, this study has provided a robust quantitative diagnostic framework for analyzing the morphological fingerprints of Yarkovsky-YORP co-evolution in asteroid families. We have learned that the choice of dependent variable, specifically $\log_{10}(\sqrt{P}/D)$, is critical for accurately representing the underlying physics of spin-orbit coupling. Crucially, our empirical findings confirm that V-shapes are dynamic structures that degrade predictably with family age, offering direct observational evidence of the long-term impact of various perturbing processes on asteroid family integrity. This framework enables a systematic classification of asteroid families based on their evolutionary state and provides a powerful new tool for probing the complex history of spin-orbit coupling, the historical efficiency of Yarkovsky and YORP effects, and the overarching dynamical evolution of the asteroid belt. Future work could build upon this foundation by incorporating observational uncertainties, exploring more complex boundary models, and conducting detailed case studies to disentangle the specific physical processes responsible for observed morphological variations.